Composite Mulanept Patterns

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Abstract

Hyperoperation transition sequences. Coordinate systems for the animations. Combinatorics of coh-formulae. Composite mulanept patterns. Boundary value problems. Uses of composite ordertype = 4 noptiles.

Keywords: Non standard tiling patterns, noptiles, nept form, mulanept form, compositions of hyperoperations, colour square diagrams, etindao numbers, information flow, type shifting patterns, non standard space filling curves,

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Part 0 Introduction

Belief in "substitution"

In maths, substitution is used in many contexts.
It allows us to ask questions such as :

$$if \;\; y = 2x + 1 \; and \;\; z = 3y^2$$

$$what \; is \; z \; in \; terms \; of \;\; x?$$

The answer is easy: plug in or substitute for y in 3(y^2) with 2x+1 and simplify.

Clarifying the terms exponential power tower, base and exponent

$$n \wedge\wedge m = \left. n^{n^{n^{\cdot^{\cdot^{\cdot n}}}}} \right\} m$$

This is an exponential power tower, but could it be called a base power tower?
In a way "yes" because the base is at the bottom and it is "n" and this base value is the only "component" of the power tower. But in another important way, it is an exponential power tower (the conventional term) because there is an implicit top-down bracketing with this notation and this implies a clear, distinctive and sequential computational direction, and therefore it is easy to see the partially evaluated power tower components becoming "exponents" for the next computation and getting progressively larger, all the way to the final computation where the base is "n" and the exponent is some huge theoretically evaluated exponent, being a power tower with :
height(exponent(n^^m)) = height(n^^m) – 1 = m – 1 .

To understand and believe in composite mulanept patterns requires what I call Belief in "substitution" where this refers to accepting the validity of 3 axioms :
Axiom of wide applicability of base exponent computation
Axiom of progressively larger exponents
Axiom of tetration with substitution

Axiom of wide applicability of base exponent computation

$$n \wedge q = n^q$$

The Axiom of wide applicability of base exponent computation says that the idea of multiplying the base "n" by itself "q" times always makes computational sense, if not in practice, then in theory.

Axiom of progressively larger exponents

$$n \wedge\wedge m = \left. n^{n^{n^{\cdot^{\cdot^{\cdot n}}}}} \right\} m$$

The Axiom of progressively larger exponents is an observation that the theoretical evaluation of a power tower starts with the top two values being base and exponent, then becoming exponent to the next base below and so on to the bottom base value. As the theoretical calculation proceeds it is observed that the exponents get progressively larger while the bases are constant.

Axiom of tetration with substitution

In order to believe in the reality of composite mulanept patterns
you need to believe in the so-called "Axiom of tetration with substitution"
This axiom says that in the expression n^^m (where n and m are natural numbers)
it does not matter how big or complicated n and/or m are ,
the expression n^^m is still a valid natural number.

The argument for believing in the Axiom of tetration with substitution
is that in the expression n^^m written as:

$$n \wedge \wedge m = \left. n^{n^{n^{\cdot^{\cdot^{n}}}}} \right\} m$$

The n's in this power tower can be any natural number, in other words, "n" may be big and complicated, but this bigness and complicatedness is hidden or encapsulated by the symbol "n".
This is true, in the sense that the power tower construction doesn't come with a proviso that the component values need to be smaller than a certain value.
Notice that "m" can also be complicated, but "m" needs to be regarded as :
a "discrete whole number amount" variable representing the height of the power tower.

The argument for disbelieving the Axiom of tetration with substitution:
is that even though "n" and "m" represent "natural" numbers, they may have such a complicated composite mulanept pattern representation so it is difficult to imagine the pattern for "n" being the componential value in an exponential power tower and it is equally difficult to imagine the pattern for "m" being a size or amount variable that describes height of power tower.
"The argument for disbelieving the Axiom of tetration with substitution" suggests that at some stage in the largeness and complexity of natural numbers the actual exponent laws make less and less sense. This may be true. Why should math laws be true for all natural numbers? For small numbers the math laws are reasonable and effective in the sciences. If mathematicians describe a result saying "for all natural numbers n : such and such" this should be overstood by the communicating mathematician AND audience to mean all natural numbers relevant and sensibly related to the context of the problem.
This is a true complex systems viewpoint about the natural numbers.

Further thoughts

In the theory of Composite Mulanept Patterns (CMP) there are 4 essential stages:

1	IML	Induced Magnitude Logic
2	IFAPL	Information Flow and Pattern Logic
3	SubL	Substitution Logic
4	(ROML	Reinterpretation of Magnitude Logic)

1 IML Induced Magnitude Logic

The origin or inspiration from exponentiation, exponent laws and theory of number bases

2 IFAPL Information Flow and Pattern Logic

The conceptual great leap forward to number representation patterns, using an optimal and consistent system, and representing distinct number classes

3 SubL Substitution Logic

For CMPs, the realisation that substitution logic occurs in stages, from simple to more complex.

4 (ROML Reinterpretation of Magnitude Logic)

This final stage is the most controversial in the theory of CMPs.
The whole idea was to move beyond magnitude logic, and see the emergence of information flow, connections and patterns as the primary logical forces for the metaclass of compositions of hyperoperations. A very advanced understanding may allow ROML questions to be posed, and possibly answered in the simpler cases.
The complexity of the problem is even more difficult than the more well known class of problems known as laddered exponents. (Conway and Guy)

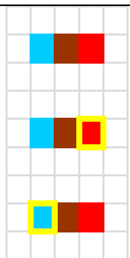
Part 1 Hyperoperations and their Compositions

Section 1.1 Starting to think about Composite Mulanept Patterns

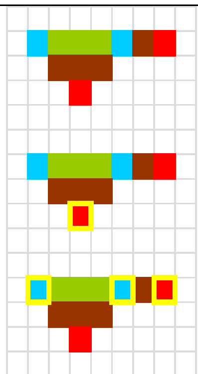
Firstly, we should be clear about n^^m versus m^^n and n^^^m versus m^^^n.
Because we can then regard the “n” as representing PVN numbers,
and the “m” as representing much bigger and more complicated numbers.

The notation followng uses Knuth arrow notation, Goodstein notation, Ept form for hyper4, Nept form for hyper5, Lanept form for hyper6, and the corresponding pure noptiles using coloured squares and coloured squares with borders for distinguishing the squares representing “m” as mentioned in the above paragraph.

Ordertype = 4

$n \wedge\wedge n = {}^{n}n = n^{n^{\cdot^{\cdot^{n}}}}\Big\}n$	
$n \wedge\wedge m = {}^{m}n = n^{n^{\cdot^{\cdot^{n}}}}\Big\}m$	
$m \wedge\wedge n = {}^{n}m = m^{m^{\cdot^{\cdot^{m}}}}\Big\}n$	

Ordertype = 5

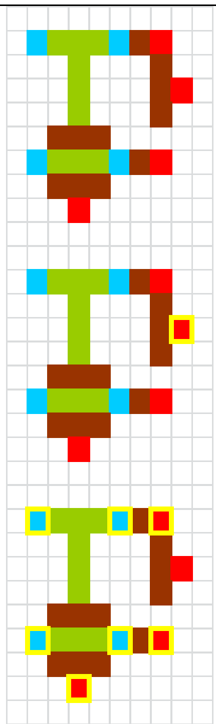
$n \wedge\wedge\wedge n = {}_{n}n = {}^{n^{\cdot^{\cdot^{n}}}}n\Big\}n = \underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\dots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n}_{n}$	
$n \wedge\wedge\wedge m = {}_{m}n = {}^{n^{\cdot^{\cdot^{n}}}}n\Big\}m = \underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\dots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n}_{m}$	
$m \wedge\wedge\wedge n = {}_{n}m = {}^{m^{\cdot^{\cdot^{m}}}}m\Big\}n = \underbrace{m^{m^{\cdot^{\cdot^{m}}}}\Big\}\dots\Big\}m^{m^{\cdot^{\cdot^{m}}}}\Big\}m}_{n}$	

Ordertype = 6

$$n\wedge\wedge\wedge\wedge n = n_n = {}_{n^{\cdot^{\cdot^{n}}}}n\}\, n = \left.\begin{matrix}\underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\ldots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n} \\ \vdots \\ \underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\ldots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n}_{n}\end{matrix}\right\}n$$

$$n\wedge\wedge\wedge\wedge m = n_m = {}_{n^{\cdot^{\cdot^{n}}}}n\}\, m = \left.\begin{matrix}\underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\ldots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n} \\ \vdots \\ \underbrace{n^{n^{\cdot^{\cdot^{n}}}}\Big\}\ldots\Big\}n^{n^{\cdot^{\cdot^{n}}}}\Big\}n}_{n}\end{matrix}\right\}m$$

$$m\wedge\wedge\wedge\wedge n = m_n = {}_{m^{\cdot^{\cdot^{m}}}}m\}\, n = \left.\begin{matrix}\underbrace{m^{m^{\cdot^{\cdot^{m}}}}\Big\}\ldots\Big\}m^{m^{\cdot^{\cdot^{m}}}}\Big\}m} \\ \vdots \\ \underbrace{m^{m^{\cdot^{\cdot^{m}}}}\Big\}\ldots\Big\}m^{m^{\cdot^{\cdot^{m}}}}\Big\}m}_{m}\end{matrix}\right\}n$$



In the “standard” geometry of the non-standard tiling patterns known as pure noptiles and composite noptiles, the ordertype = 6 Outer Seed Value block (coloured using 3 vertical brown squares for the brace and the red square for the seed value) is pushed one square further to the right. This makes the patterns extensible, less cluttered and more easy to combine together.

Section 1.2 Kinds of hyperoperation transition sequences

The wonderful world of hyperoperation transition sequences

Let’s say that n>=3.
n^^n, n^^^n, n^^^^n, n^^^^^n, n^^^^^^n, …
could be written as
n^2n, n^3n, n^4n, n^5n, n^6n, …
these are
tetration, pentation, hexation, heptation, octation, and so on, or use the terms
hyper4, hyper5, hyper6, hyper7, hyper8, …
This is the “basic hyperoperation sequence” or “pure hyperoperation transitional sequence”.
In my other papers, I referred to them as seed(n) Ackermann numbers, because they’re similar to the Ackermann numbers, but for each number in the seed(n) Ackermann number sequence, the hyperbase and hyperexponent are a constant natural number, (n). We can try to understand these numbers as **mu**lti **la**yered **n**ested **e**xponential **p**ower **t**owers. This is known as the **mulanept** form of the hyperoperations. ***Mulanept*** is the proper term for ***hyper7*** and beyond. Accurately speaking, ***hyper6*** should be described as ***Lanept***; ***hyper5*** as ***Nept; hyper4*** as ***Ept***.
We ***need*** the colored square approach to describe the ***mulanept form*** because it is mostly too messy to use a formula symbol expression from the usual typesetting software. In my papers, I use MathType quite often as it is beautiful and very user friendly and does well for ***mulanept form*** up to ***hyper7***. Mulanept form is complicated and a fairly new concept in maths, ***especially*** considering hyper7, hyper8, hyper9 and beyond. Moreover, even if it were possible to represent mulanept form with normal math symbols it would look messy and be difficult to read. The ***coloured square approach*** is the only sensible way to visualise *mulanept form*. It is not at all obvious what the pure hyperoperation transition sequence should look like ***if*** expressed in mulanept form. The mulanept form notation ***using noptiles*** for a pure hyperoperation can be enclosed by a minimal (or minimum) enclosing rectangle. In this way, we see that the ***area of the mulanept notations*** increases ***exponentially*** and the shapes alternate between approximate 2-by-1 rectangles and approximate squares, according to a prescribed folding pattern.

You could say the sequence n(^k)n (pure hyperoperation transitional sequence) is the most simple of the possible transitional sequences through the hyperoperation hierarchy. This sequence is not as informative as the non-trivial sequences.

What other ***compositions of hyperoperations*** are needed to give ***non-trivial*** transitional sequences? We could compose hyperoperations in a sequential manner where the composition schema follows the binary numbers. This is the binary-indexed (or Base2-indexed) transitional sequence.

To illustrate:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj	D5	D4	D3	D2	D1

So the Dj are binary digits and we start from 1:
1, 10, 11, 100, 101, 110, 111, 1000, 1001 and so on.

For example with the binary index value 110:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj			1	1	0

This refers to the well-defined expression: n^4(n^3n)
The bracketing is always defined as going from right to left.

For example with the binary index value 1001:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj		1	0	0	1

This refers to the well-defined expression: n^5(n^2n)
As we read these expressions from left to right we note that:
(1) the Knuth arrows are decreasing
(2) the bracketing is from right to left, in other words top-down bracketing.

The binary index sequence gives a non-trivial hyperoperation transition sequence. Looking carefully, we can notice fairly big jumps between 111…111 (m digits) and 1000…000 (m+1 digits).

One could try to smooth this index sequence by including compositions of hyperoperations indexed by digit sequences of the form 1200…00 (m digits) and 2000…00 (m digits) that are both nestled between the index numbers 111…111 (m digits) and 1000…000 (m+1 digits).

For example with 1200:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj		1	2	0	0

This refers to the well-defined expression: n^5(n^4(n^4n))

For example with 2000:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj		2	0	0	0

This refers to the well-defined expression: n^5(n^5n)

This is the idea behind the “stem2bud3 sequence” that is mostly Base2 but has two additional Base3 indices at the digit increment transitions.
The corresponding mulanept pattern animations are smoother.

Another smooth transition sequence uses Base3 as the indexing sequence.

For example with 2121:

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj		2	1	2	1

This refers to the well-defined expression: n^5(n^5(n^4(n^3(n^3(n^2n)))))

There is another important transition sequence that is simpler than the others and that uses what I call the 1020 index sequence:
1, 2, 10, 20, 100, 200, 1000, 2000, …
This produces a very interesting sequence with surprisingly smooth movements and it has the ***nice property*** that it shows in a minimal way the difference between iterates of hyper(n) and the successor hyperoperation hyper(n+1).

In summary, with the assumption of top-down bracketing starting with the smallest hyperoperation, the most natural hyperoperation transition sequences using mulanept patterns are the basic hyperoperation sequence (the “trivial” transition sequence); the 1020-indexed transition sequence; the binary-indexed transition sequence; the stem2bud3-indexed transition sequence; and the ternary-indexed transition sequence. The ternary-indexed transition sequence is useful for detailed inspection of the transitions.

Section 1.3 Coordinate systems for hyperoperation transition animations.

These coordinate systems are useful for animating the various hyperoperation transition sequences mentioned in Section 1.2.

Picture centered.

Copying the bitmaps into presentation slides, centers each of the bitmaps and so provides a “default” coordinate system.

Bottom hyperoperation, Initial Seed Value.

The index sequences from Section 1.2 have a regular nature, and although the composition of hyperoperations could be a long and complicated expression, it is still quite easy to recognise the hyperbase; the hyperexponent; the Bottom (or Base) hyperoperation and the Topmost hyperoperation and, the Initial Seed Value of the Base hyperoperation noptile. Thus, we can use the Initial Seed Value of the Base hyperoperation as the origin in a coordinate system for the animation.

For example with 2121: n^5(n^5(n^4(n^3(n^3(n^2n)))))

Hyper(n)	Hyper8	Hyper7	Hyper6	Hyper5	Hyper4
Dj		2	1	2	1

The “Base hyperoperation” is hyper7; the hyperbase is “n”
and the hyperexponent is n^5(n^4(n^3(n^3(n^2n))))

Base hyperoperation, Outer Seed Value.

We can use the Outer Seed Value of the Base hyperoperation noptile as the origin in a coordinate system for the animation.

Topmost hyperoperation, Initial Seed Value.

For example with the index 2121: n^5(n^5(n^4(n^3(n^3(n^2n)))))
The “Topmost hyperoperation” is hyper4; the hyperbase is “n”
and the hyperexponent is n^5(n^4(n^3(n^3(n^2n))))

Topmost hyperoperation, Outer Seed Value.

For example with index 2121: n^5(n^5(n^4(n^3(n^3(n^2n)))))
The “Topmost hyperoperation” is hyper4; the hyperbase is “n”
and the hyperexponent is n^5(n^4(n^3(n^3(n^2n))))

APPT Answer position power tower.

Every composite noptile has a well-defined APPT square. The APPT coordinate system has the origin where the APPT square for the composite noptile is.

Note:
Sometimes it is convenient to be more “relaxed” about the terms such as “Base hyperoperation”, “Topmost hyperoperation”, and refer to them as “hyperbase” or “hyperexponent” without causing much confusion. You could even call them “bottommost hyperoperation” and “topmost hyperoperation”.

Table showing how to colour code the formula expressions for the 4 Non Final Value (non APPT) grid coordinate systems.

S2B3	hbisv	hbosv	Heisv	heosv
1	n^^n	n^^n	n^^n	n^^n
2	n^^(n^^n)	n^^(n^^n)	n^^(n^^n)	n^^(n^^n)
10	n^3n	n^3n	n^3n	n^3n
11	n^3(n^^n)	n^3(n^^n)	n^3(n^^n)	n^3(n^^n)
12	n^3(n^^(n^^n))	n^3(n^^(n^^n))	n^3(n^^(n^^n))	n^3(n^^(n^^n))
20	n^3(n^3n)	n^3(n^3n)	n^3(n^3n)	n^3(n^3n)
100	n^4n	n^4n	n^4n	n^4n
101	n^4(n^^n)	n^4(n^^n)	n^4(n^^n)	n^4(n^^n)
110	n^4(n^3n)	n^4(n^3n)	n^4(n^3n)	n^4(n^3n)
111	n^4(n^3(n^^n))	n^4(n^3(n^^n))	n^4(n^3(n^^n))	n^4(n^3(n^^n))
120	n^4(n^3(n^3n))	n^4(n^3(n^3n))	n^4(n^3(n^3n))	n^4(n^3(n^3n))
200	n^4(n^4n)	n^4(n^4n)	n^4(n^4n)	n^4(n^4n)
1000	n^5n	n^5n	n^5n	n^5n
1001	n^5(n^^n)	n^5(n^^n)	n^5(n^^n)	n^5(n^^n)
1010	n^5(n^3n)	n^5(n^3n)	n^5(n^3n)	n^5(n^3n)
1011	n^5(n^3(n^^n))	n^5(n^3(n^^n))	n^5(n^3(n^^n))	n^5(n^3(n^^n))
1100	n^5(n^4n)	n^5(n^4n)	n^5(n^4n)	n^5(n^4n)
1101	n^5(n^4(n^^n))	n^5(n^4(n^^n))	n^5(n^4(n^^n))	n^5(n^4(n^^n))
1110	n^5(n^4(n^3n))	n^5(n^4(n^3n))	n^5(n^4(n^3n))	n^5(n^4(n^3n))
1111	n^5(n^4(n^3(n^^n)))	n^5(n^4(n^3(n^^n)))	n^5(n^4(n^3(n^^n)))	n^5(n^4(n^3(n^^n)))
1200	n^5(n^4(n^4n))	n^5(n^4(n^4n))	n^5(n^4(n^4n))	n^5(n^4(n^4n))
2000	n^5(n^5n)	n^5(n^5n)	n^5(n^5n)	n^5(n^5n)
10000	n^6n	n^6n	n^6n	n^6n
10001	n^6(n^^n)	n^6(n^^n)	n^6(n^^n)	n^6(n^^n)
10010	n^6(n^3n)	n^6(n^3n)	n^6(n^3n)	n^6(n^3n)
10011	n^6(n^3(n^^n))	n^6(n^3(n^^n))	n^6(n^3(n^^n))	n^6(n^3(n^^n))
10100	n^6(n^4n)	n^6(n^4n)	n^6(n^4n)	n^6(n^4n)
10101	n^6(n^4(n^^n))	n^6(n^4(n^^n))	n^6(n^4(n^^n))	n^6(n^4(n^^n))
10110	n^6(n^4(n^3n))	n^6(n^4(n^3n))	n^6(n^4(n^3n))	n^6(n^4(n^3n))
10111	n^6(n^4(n^3(n^^n)))	n^6(n^4(n^3(n^^n)))	n^6(n^4(n^3(n^^n)))	n^6(n^4(n^3(n^^n)))
11000	n^6(n^5n)	n^6(n^5n)	n^6(n^5n)	n^6(n^5n)
11001	n^6(n^5(n^^n))	n^6(n^5(n^^n))	n^6(n^5(n^^n))	n^6(n^5(n^^n))
11010	n^6(n^5(n^3n))	n^6(n^5(n^3n))	n^6(n^5(n^3n))	n^6(n^5(n^3n))
11011	n^6(n^5(n^3(n^^n)))	n^6(n^5(n^3(n^^n)))	n^6(n^5(n^3(n^^n)))	n^6(n^5(n^3(n^^n)))
11100	n^6(n^5(n^4n))	n^6(n^5(n^4n))	n^6(n^5(n^4n))	n^6(n^5(n^4n))
11101	n^6(n^5(n^4(n^^n)))	n^6(n^5(n^4(n^^n)))	n^6(n^5(n^4(n^^n)))	n^6(n^5(n^4(n^^n)))
11110	n^6(n^5(n^4(n^3n)))	n^6(n^5(n^4(n^3n)))	n^6(n^5(n^4(n^3n)))	n^6(n^5(n^4(n^3n)))
11111	n^6(n^5(n^4(n^3(n^^n))))	n^6(n^5(n^4(n^3(n^^n))))	n^6(n^5(n^4(n^3(n^^n))))	n^6(n^5(n^4(n^3(n^^n))))
12000	n^6(n^5(n^5n))	n^6(n^5(n^5n))	n^6(n^5(n^5n))	n^6(n^5(n^5n))
20000	n^6(n^6n)	n^6(n^6n)	n^6(n^6n)	n^6(n^6n)
100000	n^7n	n^7n	n^7n	n^7n

Section 1.4 Combinatorics of compositions of hyperoperations

This section is about the combinatorics of a small set of hyperoperations and the various ways to construct the composite noptiles.

There are 5 hyperoperations in the set {hyper4, hyper5, hyper6, hyper7, hyper8}

If we don't allow repetitions…

Number of 1 element subsets 5C1=5
Number of 2 element subsets 5C2=10
Number of 3 element subsets 5C3=10
Number of 4 element subsets 5C4=5
Number of 5 element subsets 5C5=1
The 1 element subsets can be ordered in only 1 way.
All of the distinct 2 element subsets can be ordered in exactly 2 ways.
All of the distinct 3 element subsets can be ordered in exactly 6 ways.
All of the distinct 4 element subsets can be ordered in exactly 24 ways.
The 5 element subset can be ordered in 120 ways.
Now consider the number of bracketings
1 hyperoperation corresponds with 1 kind of bracketing
2 hyperoperations corresponds with 2 kinds of bracketing
3 hyperoperations corresponds with 5 kinds of bracketing
4 hyperoperations corresponds with 14 kinds of bracketing
5 hyperoperations corresponds with 42 kinds of bracketing
So the number of patterns is

$$numpat = \sum_{i=1}^{5} ({}^5C_i) * i! * catalan_{(i+1)}$$

$$= (5*1*1) + (10*2*2) + (10*6*5) + (5*24*14) + (1*120*42)$$

$$= (5*1) + (20*2) + (60*5) + (120*14) + (120*42)$$

$$= 5 + 40 + 300 + 1680 + 5040 = 7065$$

How about if we allow repetitions of hyperoperations in our expression?

For 1 hyperoperation there are 5 ways
For 2 hyperoperations there are 5^2 ways
For 3 hyperoperations there are 5^3 ways
For 4 hyperoperations there are 5^4 ways
For 5 hyperoperations there are 5^5 ways
Just the same as before, we can consider the number of bracketings
1 hyperoperation corresponds with 1 kind of bracketing
2 hyperoperations corresponds with 2 kinds of bracketing
3 hyperoperations corresponds with 5 kinds of bracketing
4 hyperoperations corresponds with 14 kinds of bracketing
5 hyperoperations corresponds with 42 kinds of bracketing

So the number of patterns is

$$numpat = \sum_{i=1}^{5} 5^i * catalan_{(i+1)}$$

$$= (5*1) + (25*2) + (125*5) + (625*14) + (3125*42)$$

$$= 5 + 50 + 625 + 8750 + 131250 = 140680$$

In summary, using this small initial segment of the hyperoperations (from hyper4 or tetration to hyper8 or octation) there are a large number of possible compositions of hyperoperations taking into account the Catalan number different bracketing possibilities.
If we don’t allow repetitions of hyperoperators in our expression there are 7,065 possible patterns and the mulanept form noptiles would look distinctive from each other.
If we did allow repetitions of hyperoperators in our expression there are 140,680 possible patterns and the mulanept form noptiles would still be distinctive from each other.
I haven’t formally checked this distinctiveness, but I think intuitively it seems correct.
As far as whether they represent different numbers that is another (probably very difficult) question to answer. This problem is similar to the problem of the number of distinct numbers arising from all bracketings of a tetration-style power tower (sometimes referred to as laddered exponents).
So it’s possible to use the noptile pattern generating method to generate some of these patterns, whether all are possible is an unsolved problem.
To program these possible patterns would be a huge and complicated task especially because it is not clear when there would be noptile overlap problems that would require pipelines to separate out the component noptiles while retaining the appropriate connections.
But it is kind of fun to try out a small selection of these possibilities to see the resulting patterns.
The patterns then show the mulanept form of the composition of hyperoperations expression including noptile substitutions, whether they occur for the outer seedvalue only or all substitutable elements apart from the outer seedvalue (aka “almost complete” noptile substitution).
The maximum number of color ensembles required is the number needed for the bottom-up style of bracketing. This is because every noptile substitution is an “almost complete” noptile substitution.
For top-down style of bracketing only one color ensemble is needed because all the noptile substitutions occur at the unique outer seedvalue of each of the component noptiles.
You might like to see what patterns you can come up with!

Selecting examples to try out. Some suggestions.

Under the “no repetition of hyperoperations” scenario.

* Choose a number from 1 to 5
This is number of elements to take from the set {hyper4, … ,hyper8}
eg select m = 4.
** Now select 4 distinct numbers from {4,5,6,7,8}
eg select {4,6,7,8}
*** Now decide on an ordering of these numbers
eg select the order as (7,6,4,8)
From the m chosen at stage (*) we now use the mapping 1-> 1, 2-> 2, 3-> 5, 4-> 14, 5-> 42.
**** now decide on a bracketing
(as m=4, we now want to select a number from 1 to 14)
eg select 5 (we need to look at table 2 below)

Table 1

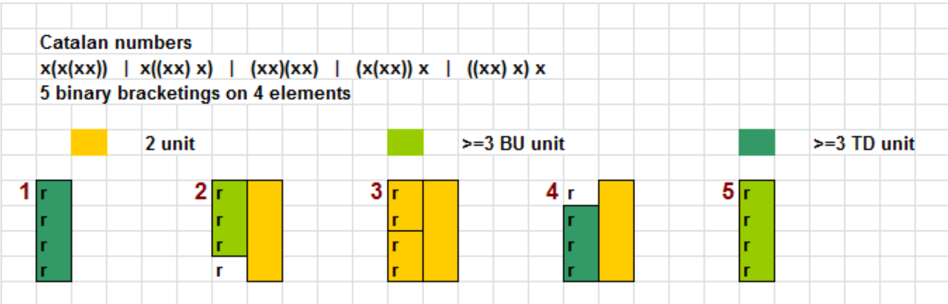


Table 2

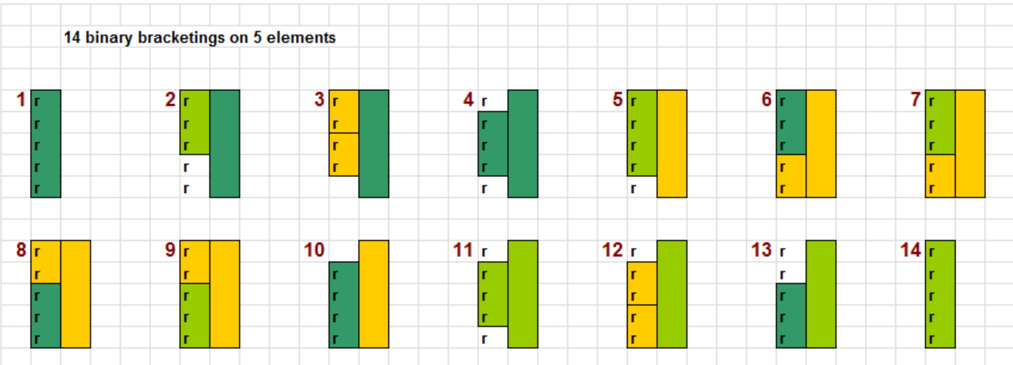
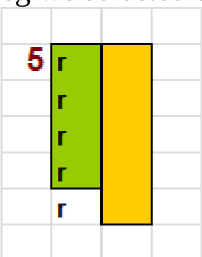


Table 3

So we selected the hyperoperations and order as: (7,6,4,8)
From the m=4 chosen at stage (*) we now use the mapping
1-> 1, 2-> 2, 3-> 5, 4-> 14, 5-> 42.
**** now decide on a bracketing
(as m=4, we now want to select a number from 1 to 14)
eg we selected 5, and then we looked at table 2 above and the pattern to use is:



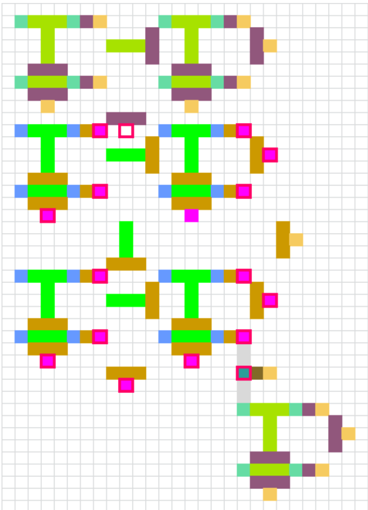
Using inline notation, this is a(((aa)a)a) [a is only a placeholder]

Now change the dummy a's to n's and use the ordering hyper(n)(7,6,4,8) to get:
n^5(((n^4n)^2n)^6n) using Knuth arrow notation (remember #Knutharrows = n – 2)

We have our selected expression to find the compound mulanept noptile for.
We can now use the N.P.G.S. or "Noptile Pattern Generating System"
to generate our pretty abstract mulanept pattern that goes with this expression.

The component patterns:
We need one each of ordertype={7,6,4,8} noptiles.
Notice that 2 extra color ensembles will be needed.

The outer seedvalue of ordertype=7 needs to be replaced by the compound mulanept noptile for the subexpression ((n^4n)^2n)^6n.
The finished pattern for n^5(((n^4n)^2n)^6n):

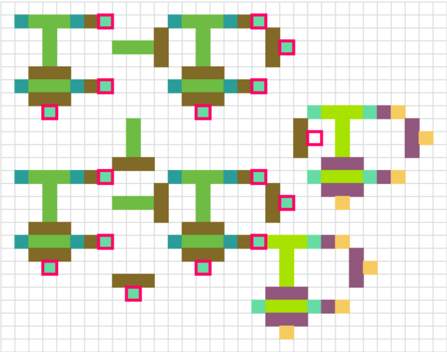


Under the "allow repetition of hyperoperations" scenario.

* Choose a number from 1 to 5
This is the number of elements to take from the set {hyper4, … ,hyper8}
eg select m = 3.
** Now select any 3 numbers (repetitions allowed) in order from {4,5,6,7,8}
eg select (6,8,6)
From the m chosen at stage (*) we now use the mapping
1-> 1, 2-> 2, 3-> 5, 4-> 14, 5-> 42.
**** now decide on a bracketing
(as m=3, we now want to select a number from 1 to 5)
eg select 3 (we need to look at table 1 above)
The selected pattern is (aa)(aa)
Now change the dummy a's to n's and use the ordering hyper(n)(6,8,6) to get:
(n^4n)^6(n^4n)
using Knuth arrow notation (remember #Knutharrows = n – 2)

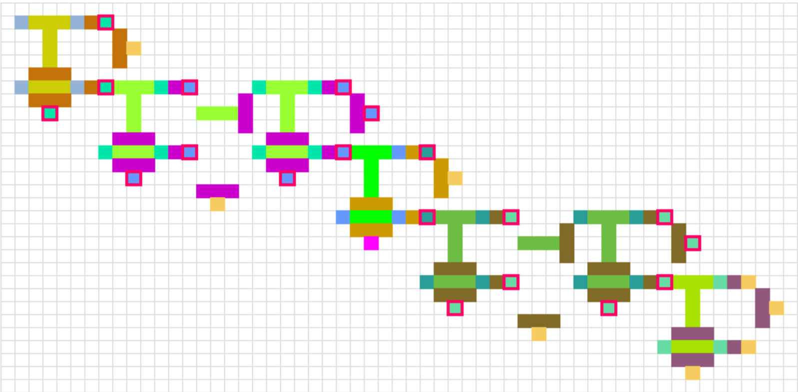
We have our selected expression to find the compound mulanept noptile for.
We need 2 ordertype=6 noptiles for the 2 (n^4n) subexpressions and 1 ordertype=8 noptile. The first of the ordertype=6 noptiles will function as an "almost complete" noptile substitution on the ordertype=8 noptile and the second will function as an "outer seed value" noptile substitution for the same ordertype=8 noptile.

The finished pattern for (n^4n)^6(n^4n):



One more example:

hyper(n)(6,7,6,7,6) using the pure bottom-up bracketing style.
Now change the dummy a's to n's and use the ordering hyper(n)(6,7,6,7,6) to get:
((((n^4n)^5n)^4n)^5n)^4n
using Knuth arrow notation (remember #Knutharrows = n – 2)
And we need 4 extra color ensembles for all the "almost complete" noptile substitutions. The finished pattern for ((((n^4n)^5n)^4n)^5n)^4n:



Section 1.5 Tables of examples for showing CMPs

You can use these tables to make your own CMPs.

Table 1

	Topdown	Bottomup
55	n^3(n^3n)	(n^3n)^3n
56	n^3(n^4n)	(n^3n)^4n
57	n^3(n^5n)	(n^3n)^5n
65	n^4(n^3n)	(n^4n)^3n
66	n^4(n^4n)	(n^4n)^4n
67	n^4(n^5n)	(n^4n)^5n
75	n^5(n^3n)	(n^5n)^3n
76	n^5(n^4n)	(n^5n)^4n
77	n^5(n^5n)	(n^5n)^5n

Table 2

	Topdown	Bottomup
567	n^3(n^4(n^5n))	((n^3n)^4n)^5n
576	n^3(n^5(n^4n))	((n^3n)^5n)^4n
657	n^4(n^3(n^5n))	((n^4n)^3n)^5n
675	n^4(n^5(n^3n))	((n^4n)^5n)^3n
756	n^5(n^3(n^4n))	((n^5n)^3n)^4n
765	n^5(n^4(n^3n))	((n^5n)^4n)^3n

(**Notice that** n^3(n^4n)=n^4[n+1])

Table 3

	((a^b)^c)^d	(a^(b^c))^d	(a^b)^(c^d)	a^((b^c)^d)	a^(b^(c^d))
567	((n^3n)^4n)^5n	(n^3(n^4n))^5n	(n^3n)^4(n^5n)	n^3((n^4n)^5n)	n^3(n^4(n^5n))
576	((n^3n)^5n)^4n	(n^3(n^5n))^4n	(n^3n)^5(n^4n)	n^3((n^5n)^4n)	n^3(n^5(n^4n))
657	((n^4n)^3n)^5n	(n^4(n^3n))^5n	(n^4n)^3(n^5n)	n^4((n^3n)^5n)	n^4(n^3(n^5n))
675	((n^4n)^5n)^3n	(n^4(n^5n))^3n	(n^4n)^5(n^3n)	n^4((n^5n)^3n)	n^4(n^5(n^3n))
756	((n^5n)^3n)^4n	(n^5(n^3n))^4n	(n^5n)^3(n^4n)	n^5((n^3n)^4n)	n^5(n^3(n^4n))
765	((n^5n)^4n)^3n	(n^5(n^4n))^3n	(n^5n)^4(n^3n)	n^5((n^4n)^3n)	n^5(n^4(n^3n))

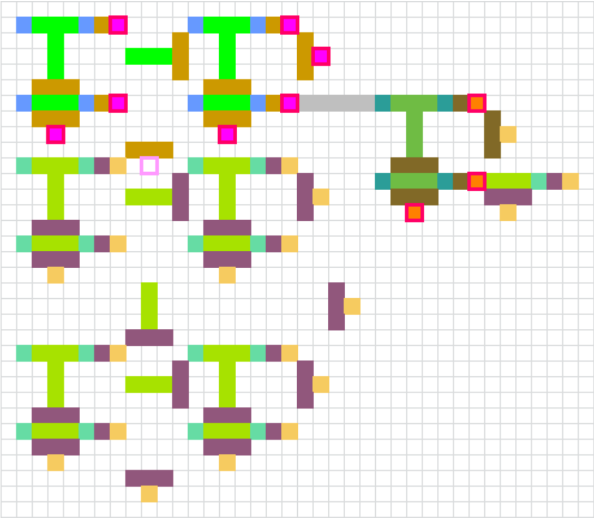
Table to check the different Catalan number kinds of Bracketing Patterns

Sum {14, 5, 4, 5, 14} = 42
Sum {42, 14, 10, 10, 14, 42} = 132
BP = Bracketing Pattern, the 42 possibles based on 5 hyperoperations or 6 elements.

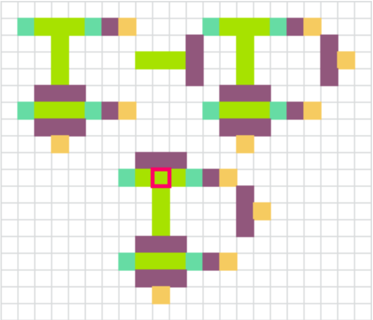
BP#	Hyper(a,b,c,d,e)	Knuth(a,b,c,d,e)	Expression
1	56565	34343	n^3(n^4(n^3(n^4(n^3n))))
2	57567	35345	n^3(n^5(n^3((n^4n)^5n)))
3	67875	45653	n^4(n^5((n^6n)^5(n^3n)))
4	65676	43454	n^4(n^3((n^4(n^5n))^4n))
5	75757	53535	n^5(n^3(((n^5n)^3n)^5n))
6	76567	54345	n^5((n^4n)^3(n^4(n^5n)))
7	55677	33455	n^3((n^3n)^4((n^5n)^5n))
8	56658	34436	n^3((n^4(n^4n))^3(n^6n))
9	67657	45435	n^4(((n^5n)^4n)^3(n^5n))
10	86865	64643	n^6((n^4(n^6(n^4n)))^3n)
11	78655	56433	n^5((n^6((n^4n)^3n))^3n)
12	66758	44536	n^4(((n^4n)^5(n^3n))^6n)
13	57566	35344	n^3(((n^5(n^3n))^4n)^4n)
14	75558	53336	n^5((((n^3n)^3n)^3n)^6n)
15	65755	43533	(n^4n)^3(n^5(n^3(n^3n)))
16	77665	55443	(n^5n)^5(n^4((n^4n)^3n))
17	66865	44643	(n^4n)^4((n^6n)^4(n^3n))
18	67757	45535	(n^4n)^5((n^5(n^3n))^5n)
19	75667	53445	(n^5n)^3(((n^4n)^4n)^5n)
20	58567	36345	(n^3(n^6n))^3(n^4(n^5n))
21	**55567**	**33345**	(n^3(n^3n))^3((n^4n)^5n)
22	**76555**	**54333**	((n^5n)^4n)^3(n^3(n^3n))
23	58567	36345	((n^3n)^6n)^3((n^4n)^5n)
24	75667	53445	(n^5(n^3(n^4n)))^4(n^5n)
25	67757	45535	(n^4((n^5n)^5n))^3(n^5n)
26	66865	44643	((n^4n)^4(n^6n))^4(n^3n)
27	77665	55443	((n^5(n^5n))^4n)^4(n^3n)
28	65755	43533	(((n^4n)^3n)^5n)^3(n^3n)
29	75558	53336	(n^5(n^3(n^3(n^3n))))^6n
30	57566	35344	(n^3(n^5((n^3n)^4n)))^4n
31	66758	44536	(n^4((n^4n)^5(n^3n)))^6n
32	78655	56433	(n^5((n^6(n^4n))^3n))^3n
33	86865	64643	(n^6(((n^4n)^6n)^4n))^3n
34	67657	45435	((n^4n)^5(n^4(n^3n)))^5n
35	56658	34436	((n^3n)^4((n^4n)^3n))^6n
36	55677	33455	((n^3(n^3n))^4(n^5n))^5n
37	76567	54345	(((n^5n)^4n)^3(n^4n))^5n
38	75757	53535	((n^5(n^3(n^5n)))^3n)^5n
39	65676	43454	((n^4((n^3n)^4n))^5n)^4n
40	67875	45653	(((n^4n)^5(n^6n))^5n)^3n
41	57567	35345	(((n^3(n^5n))^3n)^4n)^5n
42	76555	54333	((((n^5n)^4n)^3n)^3n)^3n

Three more examples

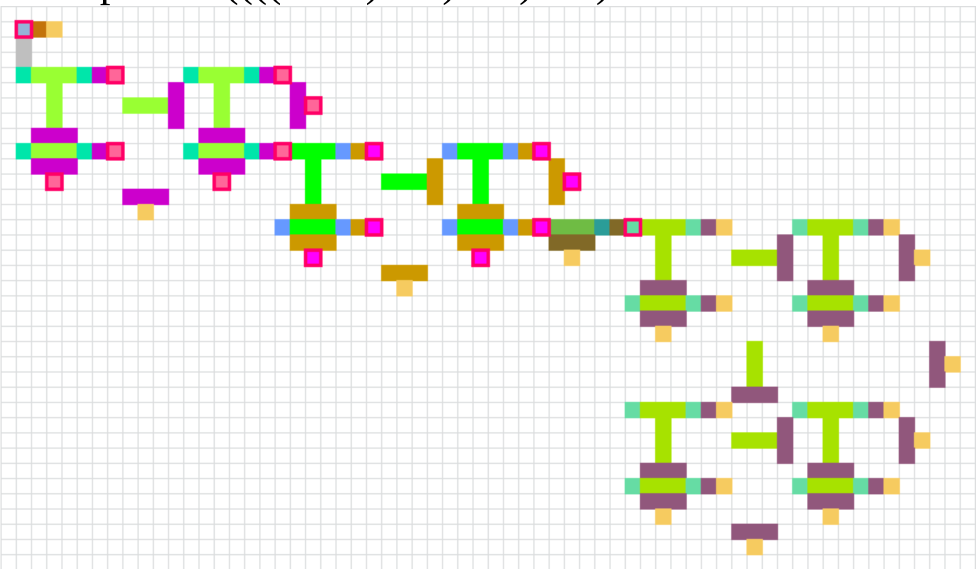
Example A: ((n^3n)^4n)^5(n^6n)



Example B: n^5(n^4n)



Example C: ((((n^6n)^3n)^5n)^5n)^2n



Part 2 Tetration and compositions of hyperoperations

Section 2.1 A theorem and proof

Conjecture about Composite Mulanept Pattern (CMP) number representations:
(mulanept = multi layered nested exponential power towers)

"Compositions of hyperoperations have a discernible twofold aspect" or
"Compositions of hyperoperations have a discernible binate nature"

1) compositions of hyperoperations can **always** be viewed as tetration (exp power tower, EPT)

2) the **dominant feature** is the information flow pattern of the CMP itself

3) CMPs show "part in whole (geometry) and whole in part (maths)" phenomena

Contextually Non-restrictive Assumption:
The ***components*** *of a* ***repnum*** *EPT (meaning all numbers in the EPT are the same) can be* ***any*** *natural number greater than or equal to 2,* ***unlike*** *digits in Base(n) Place Value Notation or MD Base(n) PVN (see relevant papers on eretrandre.org website).*

Proof: Take an arbitrary composition of hyperoperations and notice the bracketing pattern and identify the *hyperbase* and the *hyperexponent*. This will be relative to some *hyperoperation*, hyper(m), this hyperoperation always has a "standard" appearance, meaning regardless of the m value, it is a pure noptile or H-fractal-shaped optimal mulanept formula representation using coloured squares. From this Ordertype(m) Noptile (call it OMN) corresponding to hyper(m) we can always identify the Initial Seed Value and the Outer Seed Value. These values are determined by the *hyperbase* and the *hyperexponent* respectively. The composite mulanept pattern represented by composite noptiles may have grey colour pipelines that feed particular values (submulanept patterns) both into the Initial Seed Value position of OMN and the Outer Seed Value position of OMN. Now consider the answer-position-power-tower (call it APPT) of OMN. For it to be expressible as **tetration**, (hyper4, exp power tower), we need to check for *discernibility* of (1) the base components and *discernibility* of (2) the height of this power tower. For (1), the base component of APPT is the same as the base component of the Initial Seed Value of OMN. For (2), the *"height"* of the OMN, in other words, the size of the Outer Ellipsis of OMN, is well defined, and this is because it is discernible as the submulanept pattern that feeds into the Outer Seed Value position of OMN. So now, the ***height of the APPT*** is **also** discernible by following the standard information flow of the OMN noptile.

Two comments about the correct use of colour ensembles in a CMP:

[1]. With respect to OMN, the number used in the Outer Seed Value position of OMN, is only used in this one particular position, to describe the outer ellipsis of OMN. Hence, there is no need to use different colour ensembles for the different hyperexponents in the CMP.

[2]. However, for the hyberbases, in order to distinguish the changes in the types of the hyperbases, different colour ensembles ***are*** needed, ***because*** within a noptile component the hyperbase is used for all the initial seed values, all the power towers of the noptile component and all the ellipsis values of the noptile component *apart from the outer ellipsis*.

If the argument above is sound, this proves:
1) arbitrary compositions of hyperoperations can always be viewed as tetration (exponential power towers).

The proof of:
2) the dominant feature is the information flow pattern of the CMP itself

Is clear because it follows directly from the number representation technique of using colour ensembles with composite noptiles, remembering that in order to distinguish the changes in the types of the hyperbase different colour ensembles are used.

Verification of the proof:
Check the steps of the proof with various examples (with various kinds of bracketing patterns and different hyperoperation orderings) of compositions of hyperoperations, for example, by using the examples from the table: "**Table to check the different Catalan number kinds of Bracketing Patterns"** from "**Section 1.5 Tables of examples for showing CMPs".**

Part 3 Boundary value problem for some composite noptiles

Section 3.1 A result about power towers and etindao numbers.

Lemma 1 [n=2]

$$(3^3)^{(3^3)} = 3^{3.3^3} = 3^{3^3.3} = 3^{3^{3+1}}$$

$$(3^3)^{(3^3)^{(3^3)}} = (3^3)^{\left(3^{3^{3+1}}\right)} = 3^{3^{3^{3+1}}.3} = 3^{3^{3^{3+1}+1}}$$

$$(3^3)^{(3^3)^{(3^3)^{(3^3)}}} = (3^3)^{\left(3^{3^{3^{3+1}+1}}\right)} = 3^{3^{3^{3^{3+1}+1}}.3} = 3^{3^{3^{3^{3+1}+1}+1}}$$

So $(3^3)^{(3^3)}$ is an etindao number with height=3

And $(3^3)^{(3^3)^{(3^3)^{(3^3)}}}$ is an etindao number with **height=5**

And $3^{3^{3^3}}$ is a powtow with **height=4**

And clearly, $(3^3)^{(3^3)^{(3^3)^{(3^3)}}}$ is bigger than $3^{3^{3^3}}$

Lemma 2 [n=3]

$$(3^{3^3})^{(3^{3^3})} = 3^{3^3.3^{3^3}} = 3^{3^{3^3}.3^3} = 3^{3^{3^3+3}}$$

$$(3^{3^3})^{(3^{3^3})^{(3^{3^3})}} = (3^{3^3})^{\left(3^{3^{3^3+3}}\right)} = 3^{3^{3^{3^3+3}}.3^3} = 3^{3^{3^{3^3+3}+3}}$$

$$(3^{3^3})^{(3^{3^3})^{(3^{3^3})^{(3^{3^3})}}} = (3^{3^3})^{\left(3^{3^{3^{3^3+3}+3}}\right)} = 3^{3^{3^{3^{3^3+3}+3}}.3^3} = 3^{3^{3^{3^{3^3+3}+3}+3}}$$

So $(3^{3^3})^{(3^{3^3})}$ is an etindao number with height=4

And $(3^{3^3})^{(3^{3^3})^{(3^{3^3})^{(3^{3^3})}}}$ is an etindao number with **height=6**

And $3^{3^{3^{3^{3^{3}}}}}$ is a powtow with **height=6**

And $\left(3^{3^{3}}\right)^{\left(3^{3^{3}}\right)^{\left(3^{3^{3}}\right)^{\left(3^{3^{3}}\right)}}}$ is bigger than $3^{3^{3^{3^{3^{3}}}}}$

Lemma 3 [n=4]

$$\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)} = 3^{3^{3^3}.3^{3^{3^3}}} = 3^{3^{3^{3^3}}.3^{3^3}} = 3^{3^{3^{3^3}+3^3}}$$

$$\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)}} = \left(3^{3^{3^3}}\right)^{\left(3^{3^{3^{3^3}+3^3}}\right)} = 3^{3^{3^{3^{3^3}+3^3}}.3^{3^3}} = 3^{3^{3^{3^{3^3}+3^3}+3^3}}$$

$$\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)}}} = \left(3^{3^{3^3}}\right)^{\left(3^{3^{3^{3^{3^3}+3^3}+3^3}}\right)} = 3^{3^{3^{3^{3^{3^3}+3^3}+3^3}}.3^{3^3}} = 3^{3^{3^{3^{3^{3^3}+3^3}+3^3}+3^3}}$$

So $\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)}$ is an etindao number with height=5

And $\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)}}}$ is an etindao number with **height=7**

And $3^{3^{3^{3^{3^{3^{3^{3}}}}}}}$ is a powtow with **height=8**

And $\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)^{\left(3^{3^{3^3}}\right)}}}$ is **NOT** bigger than $3^{3^{3^{3^{3^{3^{3^{3}}}}}}}$!

Theorem (The powtow and etindao theorem)

For n=2 and n=3
(3^^n)^^4 > 3^^(2n)
For n>=4
(3^^n)^^4 < 3^^(2n)
This is interesting because it is saying that for n>=4,
If you have a stack of 4n 3’s (that’s a lot of 3’s) BUT grouped into 4 groups with top down bracketing,
then this is SMALLER than HALF as many 3’s (that is 2n 3’s) AND with pure top down bracketing.
Consider the “triple” (n, height((3^^n)^^4), height(3^^(2n))) where
The relevant “triples” from the examples above are:
(2,**5,4**), (3,**6,6**), (4,**7,8**).
Exercise: What is the next triple in this sequence: Is it (5,**8,10**) ??
Checking the details … The next triple in the sequence above is: (5,**8,10**)

$$\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)} = 3^{3^{3^{3^3}} . 3^{3^{3^{3^3}}}} = 3^{3^{3^{3^{3^3}}} . 3^{3^{3^3}}} = 3^{3^{3^{3^{3^3}}+3^{3^3}}}$$

$$\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)}} = \left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^{3^3}}+3^{3^3}}}\right)} = 3^{3^{3^{3^{3^{3^{3^3}}+3^{3^3}}}} . 3^{3^{3^3}}} = 3^{3^{3^{3^{3^{3^{3^3}}+3^{3^3}}}+3^{3^3}}}$$

$$\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)}}} = \left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^{3^{3^3}}+3^{3^3}}}+3^{3^3}}\right)} = 3^{3^{3^{3^{3^{3^{3^{3^3}}+3^{3^3}}}+3^{3^3}}+3^{3^3}}}$$

So $\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)}$ is an etindao number with height=6

And $\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)^{\left(3^{3^{3^{3^3}}}\right)}}}$ is an etindao number with **height=8**

So $3^{3^{3^{3^{3^{3^{3^{3^{3^3}}}}}}}}$ is a powtow with **height=10**

And $\left(3^{3^{3^{3^{3}}}}\right)^{\left(3^{3^{3^{3^{3}}}}\right)^{\left(3^{3^{3^{3^{3}}}}\right)^{\left(3^{3^{3^{3^{3}}}}\right)}}}$ is **NOT** bigger than $3^{3^{3^{3^{3^{3^{3^{3^{3^{3}}}}}}}}}$!

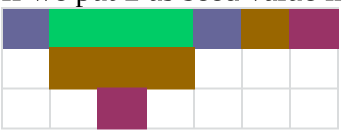
So the next triple in the sequence is: (5,**8,10**)
This suggests the general sequence pattern is :
(2,**5,4**), (3,**6,6**), (4,**7,8**), (5,**8,10**), (6,**9,12**), (7,**10,14**), …
So thinking about the inequality (3^^n)^^4 < 3^^(2n)
we see that the heights of 3^^(2n) gradually outpace the heights of (3^^n)^^4.
In the last example, the height of 3^^(2*5) is 10 and the height of (3^^5)^^4 is 8.
It is also possible to observe a regular pattern in the changes of the etindao numbers:
For the successive etindao numbers going with the sequence above, and looking from the top of the etindao number downwards, we notice the level of the first appended operations descends by 1 for successive numbers in the sequence and furthermore, the appended operation, is an addition of a powtow and this added powtow increases in height by one for the successive numbers.
That's quite an interesting and unexpected pattern to observe and this result contributes to the understanding about power towers and etindao numbers.
The main result is quite surprising because intuition might suggest that as there are twice as many 3's in the (3^^n)^^4 expression compared to the 3^^(2n) expression, that we would expect that (3^^n)^^4 should be bigger than 3^^(2n), but this is Only true for n=2 and n=3.

Section 3.2 A result about “n tetrated to n” and “3 pentated to 3”

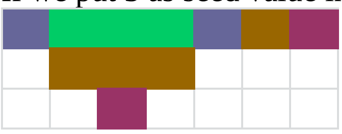
The Boundary Value Problem

This section starts off with some preliminary ideas, leading to the result.
The first non-trivial tetration number is 3^^3.
2^^2 is trivial tetration since it is same as 2^2, 2*2 and 2+2, as well as 2^^^2 etc.
3^(3^3) = 7,625,597,484,987 about 7 trillion 625 billion 600 million.
2^^2 looks like tetration so we could use the ordertype=4 pure noptile:

but 4 is clearly a small PVN number and better seen as ordertype=3 (that is to say, a PVNBase10 digit string), or possibly ordertype=2, as it is a single digit number relative to Base10.
3^^3 could be regarded as ordertype=3, as it is fairly easy to describe as a Base10 digit string, but it has 13 digits so the number of digits is ordertype=3 but not ordertype=2.
Now let’s consider the next biggest CMP number class on from  using top-down bracketing. This is clearly 
Now with 2 as seed value and base component, the number is a stack of 2’s 4 high.
In other words, 2^^(2^^2)=2^2^2^2 = 2^16 = 65,536.
If we put 2 as seed value into

 it is the same as  2^^^2=2^^2

If we put 3 as seed value into

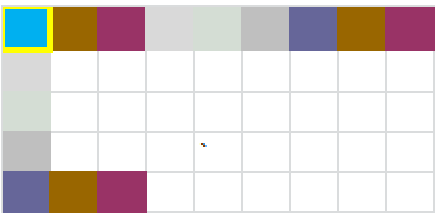
 it is the same as  3^^^3=3^^(3^^3)

Now the question is: how does the number class represented by

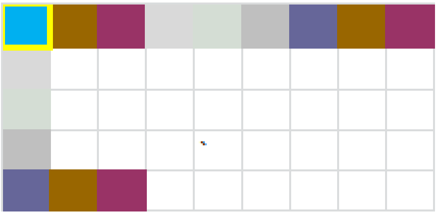
 overlap with the number class represented by  ?

We have assumed the minimal nontrivial ordertype=4 value has seedvalue=3, and this number is 3^(3^3)=3^27 as above.

If we considered [3^(3^3)] ^ [3^(3^3)] … ^ [3^(3^3)] **}** 3^(3^3) this would be :



Any number m, such that m = n ^ n … ^ n } n where 3 <= n < 3^(3^3)
could sensibly be seen as 
If we used



with seed = 2 and tetration, the number is 4^^4 and sensibly represented by

We decided that the smallest non-trivial value for 

is not 2^^(2^^2) = 2^^4 = 65,536, but is actually 3^^(3^^3) = 3^^(3^27).

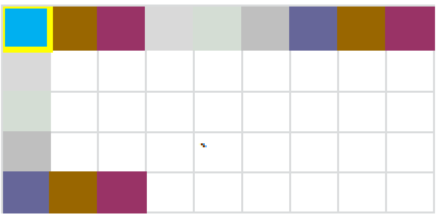
The main point we're getting to is this:

what is the boundary between  and ?

In other words, what value of n is the maximum value that satisfies the inequality: n^^n < 3^^(3^^3) ?

If we know this, then we know the boundary answer, because if we
Let N = max { n : n^^n < 3^^(3^^3) }

N^^N is suitably represented by 
What about M = (N+1)^^(N+1) ?
Then M is in the class of numbers represented by  as it is bigger than 3^^(3^^3), but it is better to regard as having the type



where we find a k value such that k^^k is close to N+1 and M is then close to (k^^k)^^(k^^k). However, working out exactly what N and k are is not easy due to it being quite a difficult arithmetic problem in the area of arithmetic computations with power towers.

Boundary value problem check.

We now consider the boundary value problem in more detail.
What value of n is the maximum value that satisfies the equation n^^n < 3^^(3^^3) ?
This question is difficult to answer exactly, but we can consider a simpler problem.
Find the Greatest Lower Bound in the natural numbers for

$$ {}^{3^n}3^n < {}^{3^{27}}3 $$

Let’s try to find Base3 component power tower Lower and Upper Bounds for power towers with the Base(3^26) component.

$$3^{26} = 3^{3.2^3+2}$$

$$(3^{26})^{(3^{26})} = 3^{3^{26}.26} = 3^{3^{3.2^3+2}.3.2^3+2}$$

$$(3^{26})^{(3^{26})^{(3^{26})}} = 3^{3^{3^{26}.26}.26} = 3^{3^{3^{3.2^3+2}.3.2^3+2}.3.2^3+2}$$

$$(3^{26})^{(3^{26})^{(3^{26})^{(3^{26})}}} = 3^{3^{3^{3^{26}.26}.26}.26} = 3^{3^{3^{3^{3.2^3+2}.3.2^3+2}.3.2^3+2}.3.2^3+2}$$

Also we have that,

$$3^{26} = 3^{3^3-1} < 3^{3^3}$$

$$(3^{26})^{(3^{26})} = 3^{3^{26}.26} = 3^{3^{3^3-1}.3^3-1} < 3^{3^{3^3}.3^3} = 3^{3^{3^3+3}}$$

$$(3^{26})^{(3^{26})^{(3^{26})}} = 3^{3^{3^{26}.26}.26} = 3^{3^{3^{3^3-1}.3^3-1}.3^3-1} < 3^{3^{3^{3^3}.3^3}.3^3} = 3^{3^{3^{3^3+3}+3}}$$

$$(3^{26})^{(3^{26})^{(3^{26})^{(3^{26})}}} = 3^{3^{3^{3^{26}.26}.26}.26} < 3^{3^{3^{3^{3^3+3}+3}+3}}$$

Combining these inequalities together we have that:

$$3^{26} = 3^{3.2^3+2} = 3^{3^3-1}$$

$$3^{3^3} < (3^{26})^{(3^{26})} < 3^{3^{3^{3^3}}}$$

$$3^{3^{3^3}} < (3^{26})^{(3^{26})^{(3^{26})}} < 3^{3^{3^{3^{3^3}}}}$$

$$3^{3^{3^{3^3}}} < (3^{26})^{(3^{26})^{(3^{26})^{(3^{26})}}} < 3^{3^{3^{3^{3^{3^3}}}}}$$

So 3^^(n+1) < (3^26)^^n < 3^^(n+3)
Substituting n = 3^26 into the inequality above …
3^^((3^26)+1) < (3^26)^^(3^26) < 3^^((3^26)+3) < 3^^(3^27)

Showing that
(3^26)^^(3^26) < 3^^(3^27)
Also, clearly
(3^27)^^(3^27) > 3^^(3^27)
This answers the question:
What value of n satisfies
max { n : (3^n)^^(3^n) < 3^^(3^27) }
The answer is n=26.
With the help of our calculators we see that :

$$3^{26} = 2{,}541{,}865{,}828{,}329$$

$$3^{27} = 7{,}625{,}597{,}484{,}987$$

And the result above can be stated as

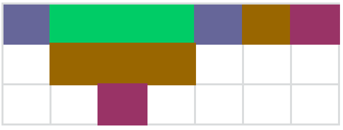
$$\left.(3^{26})^{(3^{26})^{\cdot^{\cdot^{\cdot(3^{26})}}}}\right\}3^{26} < \left.3^{3^{\cdot^{\cdot^{\cdot 3}}}}\right\}3^{27} < \left.(3^{27})^{(3^{27})^{\cdot^{\cdot^{\cdot(3^{27})}}}}\right\}3^{27}$$

Or with Knuth arrow notation,
(3^26)^^(3^26) < 3^^^3 = 3^^(3^27) < (3^27)^^(3^27)

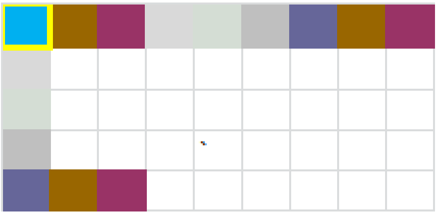
The result is suggesting that numbers with the form a^^b between 3^^3 and (3^26)^^(3^26) are sensibly regarded as tetrational numbers (the first iterate of a pure ordertype=4 noptile),

Whereas, numbers beyond 3^^^3=3^^(3^27) are sensible to regard as iterated tetration (the second iterate of an ordertype=4 noptile) or pentation.

Or



Also, while you could view (3^27)^^(3^27) as simple tetration but with a large seed value, it is sensible to regard (3^27)^^(3^27) as a composite noptile consisting of an ordertype=4 noptile with ordertype=4 hyperbase and ordertype=4 hyperexponent feeding into the hyperbase and hyperexponent positions.

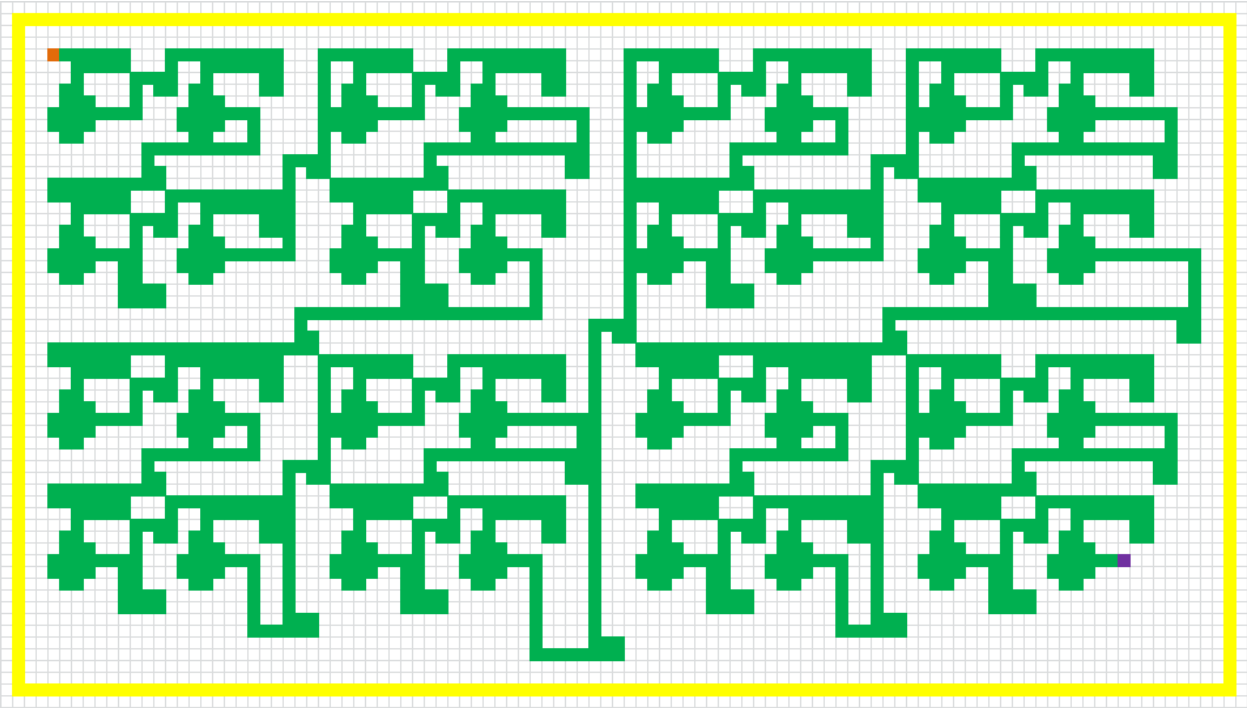


Hopefully, this gives some clarification about how the various mulanept formulae expressions with corresponding composite noptile pattern representations can represent fairly distinct number classes and shows that in the simpler cases it is possible to distinguish and describe the fuzzy regions between the different number classes.

Part 4 Non standard space filling curve

From the “default” folding pattern, fold left, up, left, up, … the pure noptiles can be constructed and it is possible to find a non standard space filling curve that starts from the Initial Seed Value (the purple square) and moves to the Answer Position Power Tower (the orange square).

Below, is a picture of this non standard space filling curve for ordertype=11.
You may observe there are 32 crossing points, where the curve self-intersects.



Part 5 List of animations

Sound pedagogy
coh=compositions of hyperoperations
easier to say coh-formulae

1) Intro to Patterns and hyperoperations	Intropatterns
2) Syntactic components	Constructnop
3) Semantics of pure coh-formulae [n(^k)n] Intro1 to semantic-syntactic correspondence the idea of thetas, msn and csds Intro2 to semantic-syntactic correspondence adapting the method for fine type distinctions	Intronoptiles PaperBexamples
4) Syntactics of folding patterns	Foldpat
5) Computational Pathways	Copathway
6) 6 useful coordinate systems picture center, APPT origin, CSBoOInSV, CSBoOOuSV, CSTmOInSV, CSTmOOuSV	picture center based; fixed origin based: APPT, CSBoOInSV, CSBoOOuSV, CSTmOInSV, CSTmOOuSV
7) 4 useful top-down transitional sequences 1020, base2, stem2bud3, base3	1020 (hyperitera), base2 (hts_b2), stem2bud3 (hts_s2b3), base3 (hts_b3)
8) Syntactics of ALL possible attachment squares in pure noptiles, useful for CMPs [see 10) & 11)]	Attsquares
9) CMPs composite patterns using hyper4 only	Iteratetra
10) 4 examples of using attachment square syntax to build composite noptiles, step by step construction, leading to coh-formula	4egcmps
11) 42 examples of coh-formulae with their csd mulanept representations	42cmps
12) 42 examples of coh-formulae with their csd mulanept representations (arty version)	42cmps arty

In total 19 animations