

Considering mdB2U3, for the transition between OT(5) and OT(6).

$$\left[ \begin{array}{c} \underbrace{11\dots 11}_{\substack{11\dots 11 \\ 11\dots 11 \\ 11\dots 11 \\ 1111111 \\ 111 \\ 11}} \end{array} \right] \left. \begin{array}{c} 1111 \\ 11 \end{array} \right\} + 1 = \underbrace{10\dots 00}_{\substack{10\dots 00 \\ 10\dots 00 \\ 10\dots 00 \\ 10000000 \\ 1000 \\ 100 \\ 11}} \left. \begin{array}{c} 1000 \\ 100 \\ 11 \end{array} \right\} 11$$

As a power tower, what is this number ?

$$11 = 2^1 + 2^0 = 3$$

$$100 = 2^2 \quad \underbrace{10\dots 00}_n = 2^{n-1}$$

$$1000 = 2^{2^2-1} \quad \underbrace{10000000}_{\substack{1000 \\ 100 \\ 11}} = 2^{2^{2^2-1}-1}$$

$$\underbrace{10\dots 00}_{\substack{10000000 \\ 1000 \\ 100 \\ 11}} = 2^{2^{2^{2^2-1}-1}-1} \quad \underbrace{10\dots 00}_{\substack{10\dots 00 \\ 10000000 \\ 1000 \\ 100 \\ 11}} = 2^{2^{2^{2^{2^2-1}-1}-1}-1}$$

$$\underbrace{10\dots 00}_{\substack{10\dots 00 \\ 10\dots 00 \\ 10000000 \\ 1000 \\ 100 \\ 11}} = 2^{2^{2^{2^{2^{2^2-1}-1}-1}-1}-1} \quad \underbrace{10\dots 00}_{\substack{10\dots 00 \\ 10\dots 00 \\ 10\dots 00 \\ 10000000 \\ 1000 \\ 100 \\ 11}} = 2^{2^{2^{2^{2^{2^{2^2-1}-1}-1}-1}-1}-1} = (A) \cong 2^{2^8} = 2^{256}$$

### Multi Dimensional Binary with Unary(3) type transitions ( mdB2U3 )

With this, type dimensional version of binary numbers, where the emphasis is on the combinatorial features of the typology, it is sensible to define Ordertype as follows:

Ordertype[0] = 0 ( by definition )  
Ordertype[1] = 1 ( by definition )  
Ordertype[2] = 1 ( there is only one non-zero, single digit number in binary )  
Ordertype[3] = { 10, 11 } ( using mdB2U3 assumption )  
Ordertype[4] = { 100, 101, 110, 111 } ( using mdB2U3 assumption )  
Ordertype[5] = { 1000 , ... , (A) - 1 } ( where " (A) " is defined as above )  
Ordertype[6] = { (A) , ... , ?? }

Converting these binary numbers to normal Base 10 ,

Ordertype[0] = 0  
Ordertype[1] = 1  
Ordertype[2] = 1  
Ordertype[3] = { 2, 3 }  
Ordertype[4] = { 4, 5, 6, 7 }  
Ordertype[5] = { 8 , ... , (A) - 1 approx 2<sup>8</sup> } ( where " (A) " is defined as above )  
Ordertype[6] = { (A) , ... , ?? }

Ordertype[4] has a surprisingly small range and

Ordertype[5] has a surprisingly large range ...

**Generalising mdB2U3, for different, regular Unary(k) type transitions.**

The values of interest are the First Values for ordertypes 3, 4, and 5.

OrderType[6] and above are more complicated and not interesting to work out.

The values for OrderType[3] are trivial. So the interesting values are OrderType[4] and OrderType[5].

Base(2)	OT(3)	B10	OT(4)	B10	OT(5)	B10
Unary(3)	10	2	100 11	4	1000 100 11	8
Unary(4)	10	2	1000 100	8	100...00 10000000 1000 100	$2^{2^{2^3-1}-1}$
Unary(5)	10	2	10000 101	16	100...00 100...00 100...00 10000 101	$2^{2^{2^{2^4-1}-1}-1}$
Unary(6)	10	2	100000 110	32	100...00 100...00 100...00 100...00 100000 110	$2^{2^{2^{2^{2^5-1}-1}-1}-1}$
Unary(7)	10	2	1000000 111	64	100...00 100...00 100...00 100...00 1000000 111	$2^{2^{2^{2^{2^{2^6-1}-1}-1}-1}-1}$
Unary(8)	10	2	10000000 1000	128	100...00 100...00 100...00 100...00 10000000 1000	$2^{2^{2^{2^{2^{2^{2^7-1}-1}-1}-1}-1}-1}$
...						

Notice that:

$$\left. \begin{array}{c} \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100000} \\ 110 \end{array} \right\} 110 < \left( \begin{array}{c} \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{10000000} \\ 1000 \\ 100 \\ 11 \end{array} \right) 11 < \left. \begin{array}{c} \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{100\dots 00} \\ \underbrace{1000000} \\ 111 \end{array} \right\} 111$$

This bizarre looking inequality is saying:

$$\text{FV}(\text{mdB2U6}, \text{OT}(5)) < \text{FV}(\text{mdB2U3}, \text{OT}(6)) < \text{FV}(\text{mdB2U7}, \text{OT}(5))$$

( where FV = First Value as defined above )

Or, the same inequality expressed in terms of Base(10) etindao numbers:

$$2^{2^{2^{2^{2^5-1-1-1-1-1}}} < 2^{2^{2^{2^{2^7-1-1-1-1-1}}} = 2^{2^{2^{2^{2^3-1-1-1-1-1}}} = 2^{2^{2^{2^{2^2-1-1-1-1-1}}} < 2^{2^{2^{2^{2^6-1-1-1-1-1}}}$$

And  $\text{FV}(\text{mdB2U3}, \text{OT}(6)) = (A)$  from above.

So this is something about the unusual world of MultiTypeDimBase(n) with regular Unary(k) type transitions, where we look at the special case of Base(2) with Unary(k) ( 3 <= k <= 8 ) type transitions.

**Generalising mdB3U3, for different, regular Unary(k) type transitions.**

The values of interest are the First Values for ordertypes 3, 4, and 5.

The values for OrderType(3) are trivial. So the interesting values are OrderType(4) and OrderType(5).

Base(3)	OT(3)	B10	OT(4)	B10	OT(5)	B10
Unary(3)	10	3	$\underbrace{100}_{10}$	9	$\underbrace{\underbrace{100000000}_{100}}_{10}$ ) <sub>10</sub>	$3^{3^2-1}$
Unary(4)	10	3	$\underbrace{1000}_{11}$	27	$\underbrace{\underbrace{100\dots 00}_{100\dots 00}}_{1000}$ ) <sub>11</sub>	$3^{3^{3^3-1}-1}$
Unary(5)	10	3	$\underbrace{10000}_{12}$	81	$\underbrace{\underbrace{100\dots 00}_{100\dots 00}}_{10000}$ ) <sub>12</sub>	$3^{3^{3^{3^4-1}-1}-1}$
Unary(6)	10	3	$\underbrace{100000}_{20}$	243	$\underbrace{\underbrace{100\dots 00}_{100\dots 00}}_{100000}$ ) <sub>20</sub>	$3^{3^{3^{3^{3^5-1}-1}-1}-1}$
Unary(7)	10	3	$\underbrace{1000000}_{21}$	729	$\underbrace{\underbrace{100\dots 00}_{100\dots 00}}_{1000000}$ ) <sub>21</sub>	$3^{3^{3^{3^{3^{3^6-1}-1}-1}-1}-1}$
Unary(8)	10	3	$\underbrace{10000000}_{22}$	2187	$\underbrace{\underbrace{100\dots 00}_{100\dots 00}}_{10000000}$ ) <sub>22</sub>	$3^{3^{3^{3^{3^{3^{3^7-1}-1}-1}-1}-1}-1}$
...						

From the examples in the table above we can conjecture the general form. We can see the pattern clearly, and the problem is regular and clear (we note that n in Base(n) is 10, but n can be described in normal Base(10) as "n" with the usual Base(10) PVN assumption) :

Given Base(n) with regular Unary(k) type transitions, then the First Values for the OrderTypes 3, 4 and 5, described in normal Base(10) are:

$$\text{OT}(3): n \quad \text{OT}(4): n^{k-1}$$

$$\text{OT}(5): \left( n^{n^{n^{n^{n^{k-1}-1}-1}-1}-1}-1} \right) \text{with } (k-1) \text{ n's}$$

This formula is for *regular* Unary(k) type transitions, where each transition length is the same. Another variation is where the OT(4) transition length is different from the OT(5) transition length. Using Unary(k1) for OT(4) and Unary(k2) for OT(5), the generalised formula for the First Value of OrderType(5) is as follows :

$$\left( n^{n \cdot n^{k_1-1-1-1} \dots -1} \right) \text{with } (k_2 - 1) \text{ } n \text{'s}$$

Some examples follow to illustrate this result, varying the Base, k1 and k2 parameters :

Base, k1, k2	OT(5)	Equivalent Base(10) etindao number
3, 3, 3	$\left( \underbrace{10\dots 00}_{100} \right)_{10}$	$\left( 3^{3^2-1} \right)$
10, 3, 3	$\left( \underbrace{10\dots 00}_3 \right)_3$	$\left( 10^{10^2-1} \right)$
3, 8, 3	$\left( \underbrace{10\dots 00}_{10000000} \right)_{10}$	$\left( 3^{3^7-1} \right)$
10, 8, 3	$\left( \underbrace{10\dots 00}_8 \right)_3$	$\left( 10^{10^7-1} \right)$
3, 3, 4	$\left( \underbrace{10\dots 00}_{100} \right)_{11}$	$\left( 3^{3^{3^2-1}-1} \right)$
10, 3, 4	$\left( \underbrace{10\dots 00}_3 \right)_4$	$\left( 10^{10^{10^2-1}-1} \right)$
3, 8, 4	$\left( \underbrace{10\dots 00}_{10000000} \right)_{11}$	$\left( 3^{3^{3^7-1}-1} \right)$
10, 8, 4	$\left( \underbrace{10\dots 00}_8 \right)_4$	$\left( 10^{10^{10^7-1}-1} \right)$