Integer Towers and Super-roots are Disjoint

Thesis:

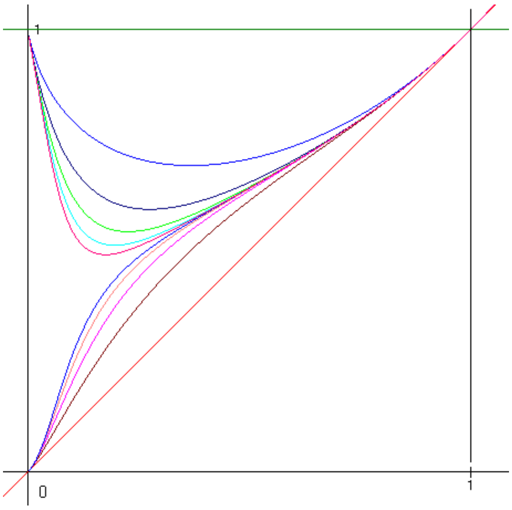
The domain and ranges of towers with integer super-exponents and of super-roots with integer root-order are disjoint, in the domain $0 < \{x, y\} < 1$.

Elements for a Demonstration

(1) – Data obtained by the calculation of integer super-exponent towers.

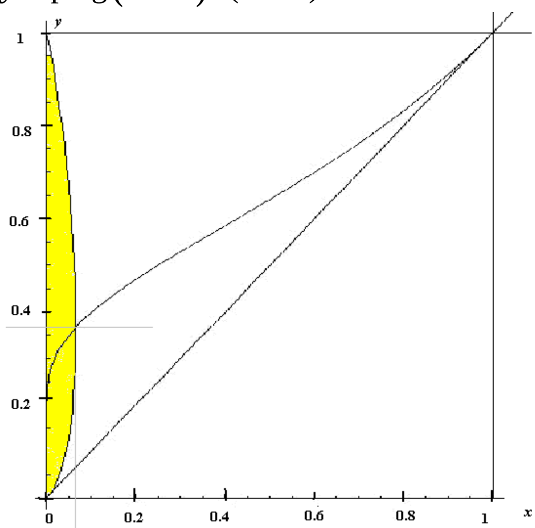
We already know that the tower functions, calculated for integer super-exponents, i.e.:

$y = {}^{n}x$, with n integer > 0, give the following plots, for n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10:



The curves with even super-exponents are passing by point (0,1) and those with even super-exponents by point (0,0). In the middle, we have the plot of:

$y = {}^{\infty}x$, given by: $y = \text{plog}\left(-\ln x\right)/(-\ln x)$



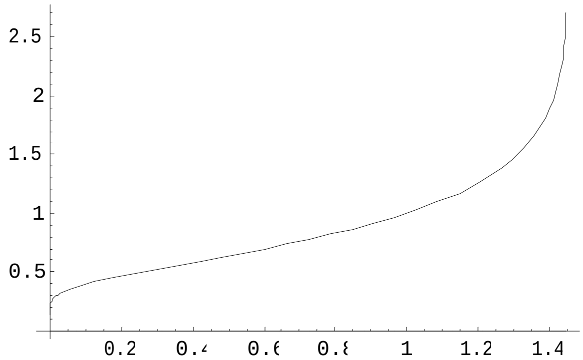
The "yellow zone", carefully avoided by the integer super-exponent towers can be interpreted with:

$$x = \kappa \cdot \left(y^{-y} - 1\right), \quad \text{i.e.: } y = \sqrt{\frac{1}{1 + x/\kappa}}, \quad \text{with: } \kappa = \frac{e^{-e}}{e^{1/e} - 1} = 0.148398483..$$

NB: We gave these heuristic formulas in NKS III. They respect the coordinates of the experimental apex point $x_1 = e^{-e} = 0.065988036..$, $y_1 = e^{-1} = 0.367879441..$ and are built with functions similar to those appearing in the further developments. Nevertheless, they need a careful discussion.

Now, since:

$y = {}^{0}x = 1$ and $y = {}^{1}x = x$ and $y = {}^{\infty}x$ is given by:



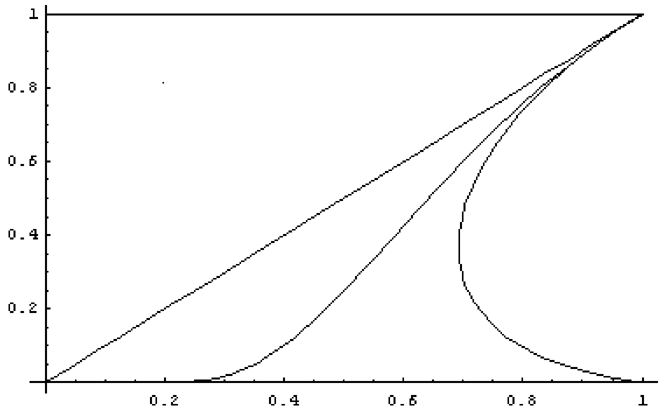
we may conclude that the plots of the integer towers is confined to the upper (0,0), (0,1), (1,1) triangle.

(2) – Possible behaviour of the integer super-roots obtained by graphical inversion

Let us start by observing that we have:

$$y = {}^{\infty}\sqrt{x} = \sqrt[x]{x} \quad \text{and} \quad y = {}^{2}\sqrt{x} = \ln x / \mathrm{plog}(\ln x)$$

which are shown in the following plot (together with $y = x$):



It evident that the following functions are the respective mutually inverse:

$$y = {}^{1}x = x \quad \rightleftharpoons \quad y = {}^{1}\sqrt{x} = x$$

$$y = {}^{2}x \quad \rightleftharpoons \quad y = {}^{2}\sqrt{x}$$

$$y = {}^{3}x \quad \rightleftharpoons \quad y = {}^{3}\sqrt{x}$$

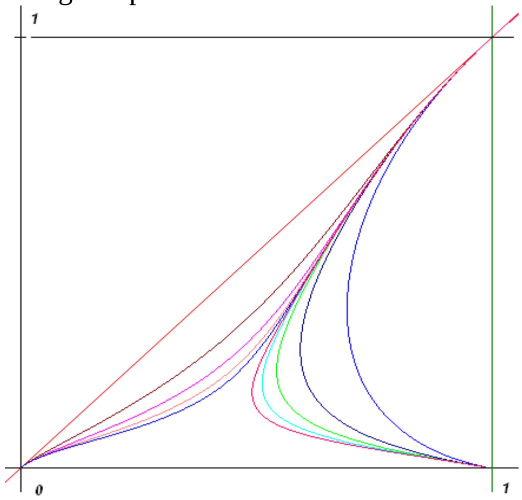
....

$$y = {}^{\infty}x \quad \rightleftharpoons \quad y = {}^{\infty}\sqrt{x}$$

We could then imagine to obtain the graphs of the integer super-roots by a systematic graphical inversion of the integer towers (symmetrically in respect of the principal diagonal).

In analogy with what was observed in section (1), we may conclude that the plots of the integer order super-root are confined into the lower (0,0), (1,0), (1,1) triangle.

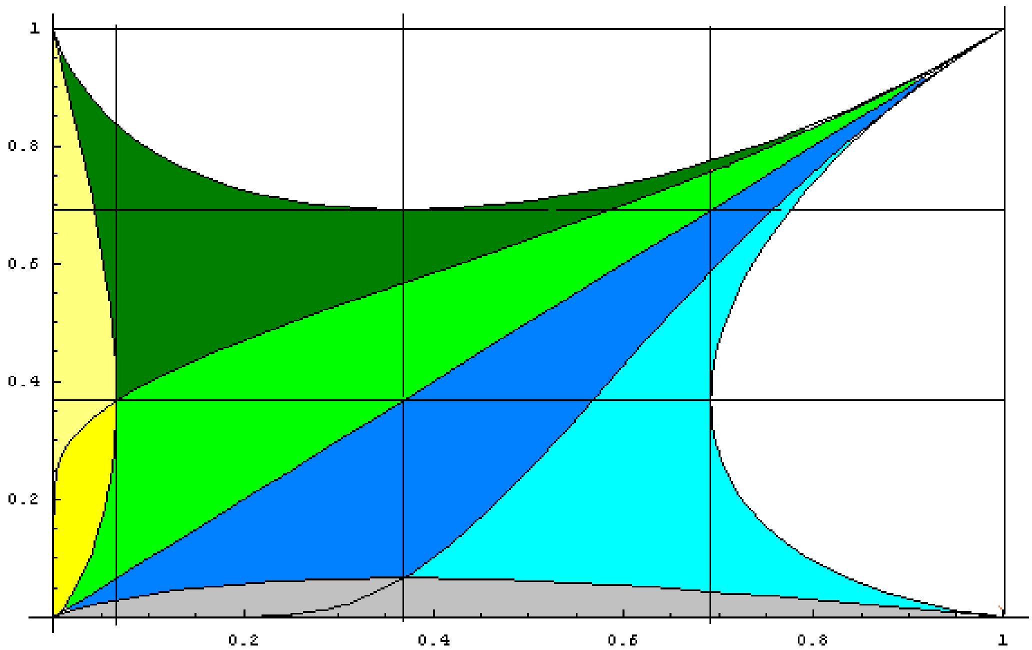
The possible behaviour of the integer super-roots could be then be as follows:



where the plots of the odd-order roots passing by point (0,0) and those with even order by point (1,0).

(3) – <u>Tentative general display of the tower and super-root domains</u>

The general display of the domains of existence of the integer tower and super-root curves can be shown as follows:



The domains of the various functions should be as follows:

- dark green area - the even integer towers $y = {}^n x$, for $n = 2, 4, 6, 8, \ldots \infty$;
- light green area - the odd integer towers $y = {}^n x$, for $n = 1, 3, 5, 7, \ldots \infty$;
- dark blue area - the odd integer super-roots $y = {}^n\!\!\sqrt{x}$, for $n = 1, 5, 7, 9, \ldots \infty$;
- light blue area - the even integer super-roots $y = {}^n\!\!\sqrt{x}$, for $n = 2, 4, 6, 8, \ldots \infty$.

NB – The "yellow zone" is an area of non-accessibility for the plots of the integer towers. For bases $x < e^{-e}$ (the apex extremum) the infinite towers ($y = {}^\infty x$) diverge, because they oscillate. The elongation being given by the perimeter of the "zone". The "grey zone" should be analyzed.

(4) – Remarkable interval limits

The examination of the last figure puts in evidence a set of remarkable interval limits, with the following main meanings:

- $e^{-e} = 0.065988036..$ the first coordinate of the apex of the "yellow zone", lower limit of the Euler's range of convergence of the infinite towers;
- $e^{-1} = 0.367879441..$ the x coordinate of minimum of the "squared" tower and the second coordinate of the apex of both the "yellow zone" and of the square super-root;
- $e^{-1/e} = 0.69220628..$ the minimum range of the square super-root ;
- $e^{0} = 1$ a remarkable frontier point for the base of the tetrational function.

To these remarkable numbers, we may add the following, for symmetrical reasons :

- $e^{1/e} = 1.444667861..$ the upper limit of convergence of the infinite towers, upper limit of the Euler's range:
- $e^{1} = 2.718281828..$ the base of the natural logarithms, also remarkable base for the tetrational function and for the super-logarithm;
- $e^{e} = 15.426224..$ KAR remarkable number ... (I don't remember why).

(5) – Possible conclusions

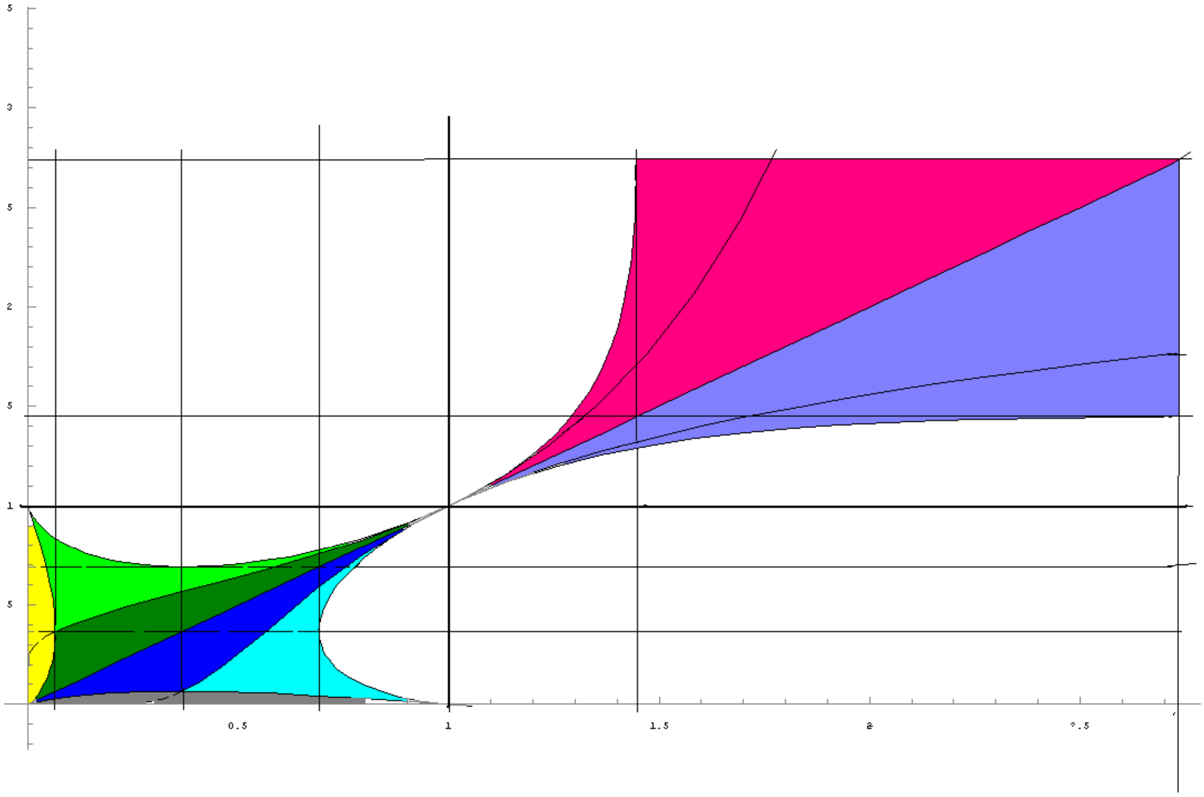
Through the examination of the last figure, we might have enough elements for a good demonstration. The towers and super-roots are disjoint. This cuts all the wrong speculations around that.

Concerning the last figure: can we detect a projective mapping? What do you think of that?

Thank you for your attention.

GFR
Le Cannet
2007-09-30

Addendum.   Extension of domains and ranges for {*x*, *y*} > 1.   2008-01-13.



GFR – 2008-01-13