

No magic, indeed. But, don't lose the faith. In fact, in Formula 1, by supposing  ${}^x e$  smooth, let's try very simply by putting:

$$\frac{d}{dx}({}^{x-1}e) = \frac{d}{dx}(\ln({}^x e))$$

i.e.:

$$[{}^{x-1}e]' = \frac{1}{{}^x e} \cdot [{}^x e]'$$

therefore:

$$\boxed{{}^x e = [{}^x e]' / [{}^{x-1}e]'}$$

In Formula 2, under the same hypothesis and taking into account Formula 1, we have:

$${}^{x+1}e \cdot {}^x e = [{}^{x+1}e]' / [{}^{x-1}e]'$$

i.e.:

$$({}^x e) \cdot e^{({}^x e)} = [{}^{x+1}e]' / [{}^{x-1}e]'$$

therefore:

$$\boxed{{}^x e = \text{plog} \left\{ [{}^{x+1}e]' / [{}^{x-1}e]' \right\}}$$

where **plog** is the product logarithm, or the ProductLog[] operator, in Mathematica.

**GFR**