INTRODUCTION TO HYPEROPERATIONS

By Alister Wilson

THE EXPONENT LAWS.

The natural numbers are

{ 1, 2, 3, … }

Let a, b, c be any natural numbers

Here are the exponent laws

$$a^{b+c} = a^b \cdot a^c$$

$$(a^b)^c = a^{b \cdot c}$$

$$(a \cdot b)^c = a^c \cdot b^c$$

(+) , (*) , AND (^)

Consider the numbers 2 and 3 and the " operations " of addition (+) multiplication (*) exponentiation (^) and " exponential towers " of height 3

(+), (*), AND (^)

2+2=4	2+3=5	3+2=5	3+3=6
2*2=4	2*3=6	3*2=6	3*3=9
2^2=4	2^3=8	3^2=9	3^3=27

EXPONENTIATION TOWERS OF HEIGHT 3

The " exponentiation towers " of height 3 use the " power tower " notation

$$2^{2^2} = 2^4 = 16 \qquad 2^{2^3} = 2^8 = 256$$

$$2^{3^2} = 2^9 = 512 \qquad 2^{3^3} = 2^{27} = 134{,}217{,}728$$

$$3^{2^2} = 3^4 = 81 \qquad 3^{2^3} = 3^8 = 6{,}561$$

$$3^{3^2} = 3^9 = 19{,}683 \qquad 3^{3^3} = 3^{27} = 7{,}625{,}597{,}484{,}987$$

Out of these 8 expressions, 2 of them are “ simpler ” than the others because they have only one repeated digit in the power tower

$$2^{2^2} = 2^4 = 16$$

$$3^{3^3} = 3^{27} = 7{,}625{,}597{,}484{,}987$$

We can use a slightly different notation for these kind of numbers

$$^{3}2 = 2^{2^{2}}$$

$$^{3}3 = 3^{3^{3}}$$

$$^{3}4 = 4^{4^{4}}$$

And these power towers can be made “ higher ” by increasing the hyperexponent

$$ {}^{4}2 = 2^{2^{2^{2}}} $$

$$ {}^{4}3 = 3^{3^{3^{3}}} $$

$$ {}^{4}4 = 4^{4^{4^{4}}} $$

This is known as “ tetration ” (hyper4), the operation after exponentiation (hyper3)

Consider the following numbers :

$$^{2}2 = 2^{2} \qquad ^{3}3 = 3^{3^{3}} \qquad ^{4}4 = 4^{4^{4^{4}}}$$

We see the “ hyperbase ” is the same as the “ hyperexponent ”

We use the normal algebra convention that an arbitrary number is “ n ”

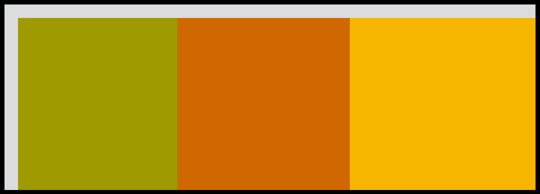
$$ {}^{n}n = n^{n^{n^{\cdot^{\cdot^{n^{n}}}}}} \left.\vphantom{\Bigg|}\right\} n = n^{n^{\cdot^{\cdot^{n}}}} \left.\vphantom{\Big|}\right\} n = n^{\cdot^{\cdot^{n}}} \left.\vphantom{\Big|}\right\} n = \theta \,\} n \quad where \quad \theta = n^{n^{\cdot^{\cdot^{n}}}} $$

So now we can colour map the **3** symbols in

$\theta \} n$

to **3** different colour squares

We could use the colours



That's the basic idea that proves to be useful for representing the hyperoperations beyond exponentiation.

Hyperoperations can be described in terms of " multi - layered nested exponential power towers "

… known as " mulanept " form.

For example, the next hyperoperation after “ tetration ” is “ pentation ”

tetra is a Greek prefix for 4

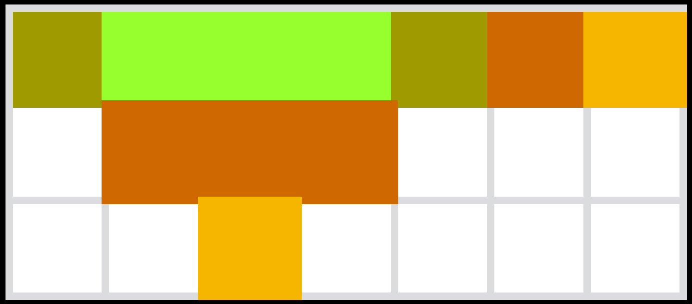
penta is a Greek prefix for 5

$$ {}_{3}3 = 3^{\cdot^{\cdot^{3}}}\Big\}3^{\cdot^{\cdot^{3}}}\Big\}3 = \underbrace{3^{\cdot^{\cdot^{3}}}\Big\}3^{\cdot^{\cdot^{3}}}\Big\}3}_{3} $$

$$ {}_{4}4 = 4^{\cdot^{\cdot^{4}}}\Big\}4^{\cdot^{\cdot^{4}}}\Big\}4^{\cdot^{\cdot^{4}}}\Big\}4 = \underbrace{4^{\cdot^{\cdot^{4}}}\Big\}4^{\cdot^{\cdot^{4}}}\Big\}4^{\cdot^{\cdot^{4}}}\Big\}4}_{4} $$

$$ {}_{n}n = \underbrace{n^{\cdot^{\cdot^{n}}}\Big\}\ldots\Big\}n^{\cdot^{\cdot^{n}}}\Big\}n}_{n} $$

And a natural colour square diagram for pentation looks like this



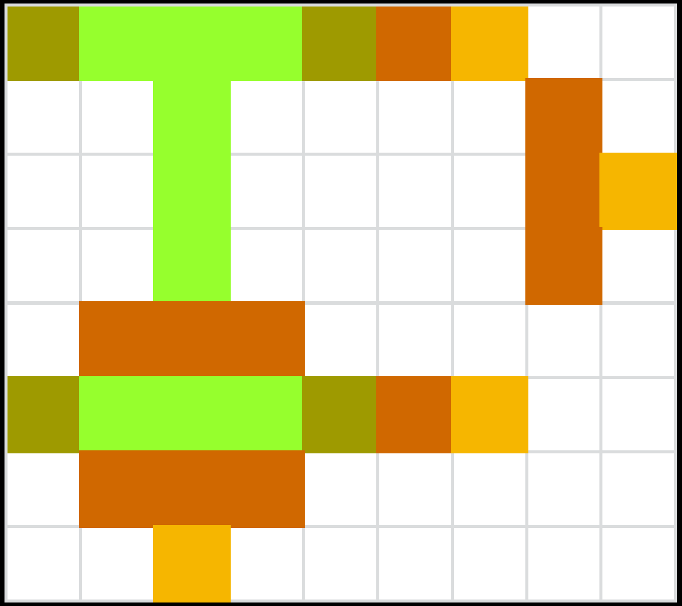
This colour mapping uses a natural " minimal symbolic notation " to make the information flow patterns clearer

With some practice, they are easy to read and recognise as " mulanept " number representations

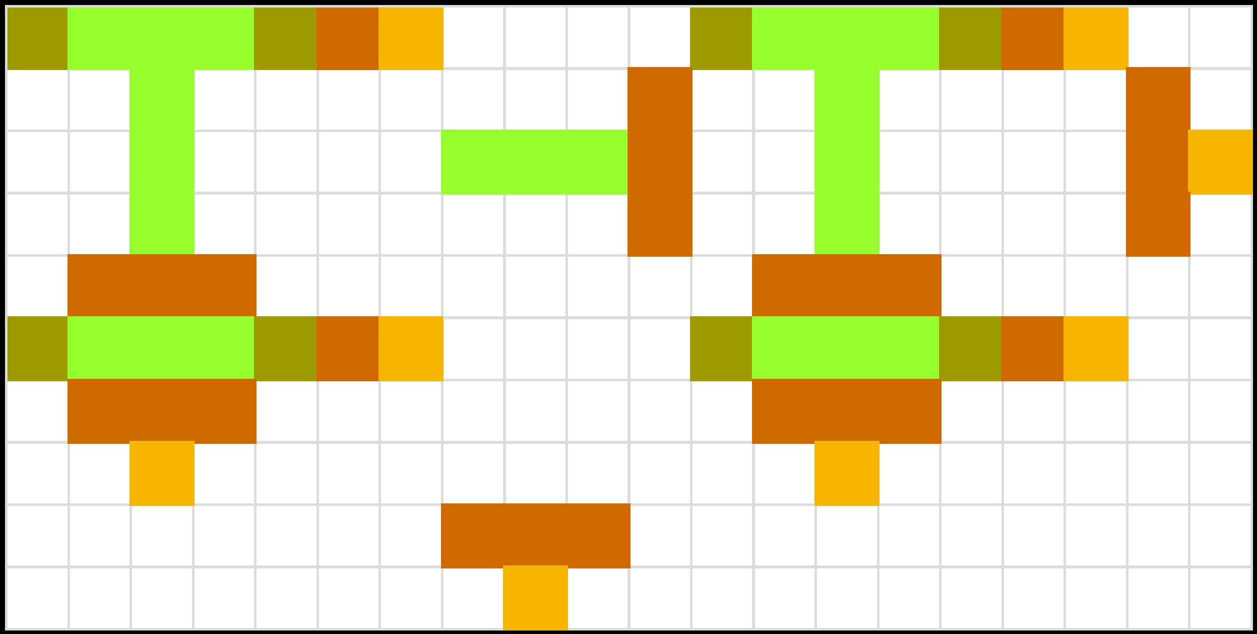
HYPER4 (ORDER TYPE = 4)

HYPER5 (ORDER TYPE = 5)

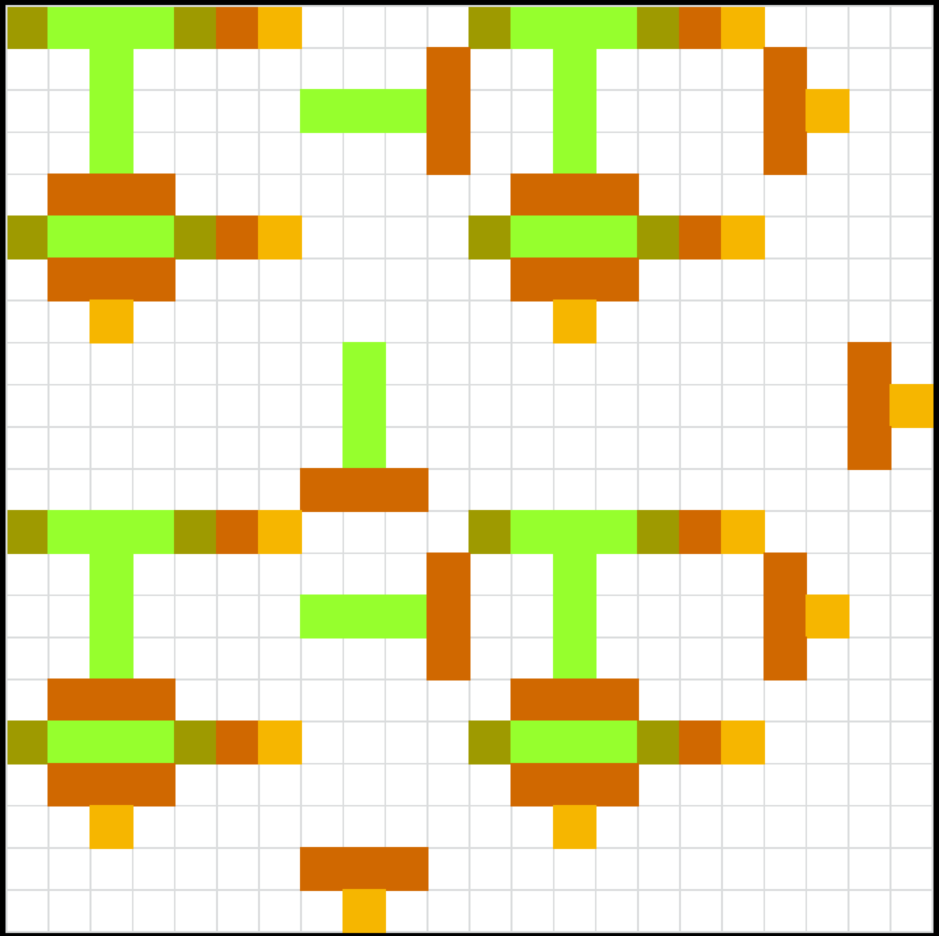
HYPER6 (ORDER TYPE = 6)



HYPER7 (ORDER TYPE = 7)



HYPER8 (ORDER TYPE = 8)



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