# <u>Tetration versus Tetraroot</u>

Yes, I agree with the UVIV's conclusions, i.e. that we must have:

$$ {}^{1/n}\left({}^{n}x\right) \neq x $$

which means that: $\quad {}^{n}x \neq {}^{1/n})\overline{x} \quad$ [$x$-tower-$n$ $\neq$ tetraroot of $x$, order $1/n$]

or, also: $\quad \boxed{{}^{1/n}x \neq {}^{n})\overline{x}} \quad$ [${}^{1/n}x \neq$ tetraroot of $x$, order $n$]

Indeed so!

In fact, let us consider the following data, for any real $b > 0$:

$$ {}^{1}b = \log_b b^b = b $$

$$ {}^{0}b = \log_b b = 1 $$

$$ {}^{-1}b = \log_b 1 = 0 $$

$$ {}^{-2}b = \log_b 0 = -\infty $$ .... hops, sorry!

And then, keeping in mind that the infinite tetraroot (super-root) of $b$ is equal to $\sqrt[b]{b}$ , let us take into consideration the following expressions:

$$ \lim_{n\to+\infty} {}^{1/n}b = {}^{0}b = 1 $$

$$ \lim_{n\to+\infty} {}^{n})\overline{b} = \sqrt[b]{b} = f(b) $$

with, in general, $f(b) \neq 1$, except for $b = 1$ and $b = +\infty$ (... sorry again). Therefore, in general, we shall indeed have:

$$ {}^{1/n}b \neq {}^{n})\overline{b} $$

except in two singular points ($b = 1$ and $b \to +\infty$). In particular for $n \to +\infty$, we have:

$$ {}^{0}b = 1 \neq {}^{\infty})\overline{b} = \sqrt[b]{b} $$

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