The Derivative of E tetra x

First off, a few notes on the notation used by this paper. Also, as this is my first time trying anything like this, I apologize for any formatting errors or obvious math mistakes made here.

$T[x] = n \text{ tetra } x$

$TE[x] = E \text{ tetra } x$

$D[f[x], x] = f'[x]$

Ok, to business. To find the derivative, let's start with a basic identity.

$TE[x] == TE[x]$

Taking the natural log of both sides gives

$Log[TE[x]] == Log[TE[x]]$

One of the tetration identies is

$Log[T[x]] == T[x-1] * Log[n]$

Or, using E as the base:

$Log[TE[x]] == TE[x-1]$

As a result,

$D[Log[TE[x]], x] == TE'[x-1]$

$$\frac{TE'[x]}{TE[x]} == TE'[x-1]$$

$TE'[x] == TE[x] * TE[x-1]$

Thus we have a recurrence relation for the derivative. This can be continued further.

$TE'[x] == TE[x] * TE[x-1] * TE'[x-2]$

$TE'[x] == TE[x] * TE[x-1] * TE[x-2] * TE'[x-3]$

This is, so far, based entirely off of http://math.eretrandre.org/tetrationforum/showthread.php?tid=47. However, I took it a bit further, realizing that (for whole numbers, anyway), you could find the product of the TE terms:

$TE'[x] == TE'[0] \prod_{k=0}^{x} TE[x-k]$

I don't know enough about partial products to be able to know what to do in the case of non-integers here, but I figured that figuring out a general formula even only for integer values of x would be useful, so I tried solving the product the same way (more or less) you would solve a sum of powers:

$Product == \prod_{k=0}^{x} TE[x-k]$

$$\text{Product} == \text{TE}[x] * \text{TE}[x-1] * \text{TE}[x-2] * \text{TE}[x-3] \ldots * \text{TE}[x-k]$$

$$E^{\text{Product}} == \text{TE}[x+1] * \text{TE}[x] * \text{TE}[x-1] * \text{TE}[x-2] \ldots * \text{TE}[x-k+1]$$

$$E^{\text{Product}} * \text{TE}[x-k] == \text{TE}[x+1] * \text{TE}[x] * \text{TE}[x-1] * \text{TE}[x-2] \ldots * \text{TE}[x-k+1] * \text{TE}[x-k]$$

$$E^{\text{Product}} * \text{TE}[x-k] == \text{TE}[x+1] * \text{Product}$$

Since

$$x - k == 0,$$

$$\text{TE}[x-k] = 1$$

As a result,

$$E^{\text{Product}} == \text{TE}[x+1] * \text{Product}$$

Now, we rearrange the equation a bit.

$$\text{Log}\left[E^{\text{Product}}\right] == \text{Log}[\text{TE}[x+1]] * \text{Log}[\text{Product}]$$

$$\text{Product} == \text{TE}[x] * \text{Log}[\text{Product}]$$

$$\text{Product} == \text{Log}\left[\text{Product}^{\text{TE}[x]}\right]$$

$$E^{\text{Product}} == \text{Product}^{\text{TE}[x]}$$

Substituting into the above equation gives

$$\text{TE}[x+1] * \text{Product} == \text{Product}^{\text{TE}[x]}$$

$$\text{TE}[x+1] == \text{Product}^{\text{TE}[x]-1}$$

$$\text{Product} == \text{TE}[x+1]^{\frac{1}{\text{TE}[x]-1}}$$

Now that there is a formula for the product:

$$\text{TE}'[x] == \text{TE}'[0] * \text{TE}[x+1]^{\frac{1}{\text{TE}[x]-1}}$$

Sadly, this can be trivially proven not to work. If x=2, and with the derivative recurrence equation listed above,

$$\text{TE}'[x] = \text{TE}[x] * \text{TE}'[x-1]$$

$$\text{TE}'[0] * \text{TE}[x+1]^{\frac{1}{\text{TE}[x]-1}} == \text{TE}[x] * \text{TE}'[0] * \text{TE}[x]^{\frac{1}{\text{TE}[x-1]-1}}$$

$$\text{TE}[x+1]^{\frac{1}{\text{TE}[x]-1}} == \text{TE}[x] * \text{TE}[x]^{\frac{1}{\text{TE}[x-1]-1}}$$

$$\text{TE}[3]^{\frac{1}{\text{TE}[2]-1}} == \text{TE}[2]^{\frac{\text{TE}[1]}{\text{TE}[1]-1}}$$

$$2.917275 \neq 73.71885$$

So after all that, it turns out not to be true. What I can't figure out is why. I'm hoping you guys could show me what's wrong with this derivation. Thanks in advance for your help.